## Smooth Bend

## Circular Cross-Section <br> (Pipe Flow - Guide)



## Model description:

This model of component calculates the head loss (pressure drop) of a smoothly curved bend whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the bend.

## Model formulation:

Hydraulic diameter ( $m$ ):
$d_{\mathrm{h}}=\frac{2 \cdot w \cdot h}{w+h}$

Cross-section area ( $m^{2}$ ):

$$
\mathrm{A}=w \cdot h
$$

Mean velocity ( $\mathrm{m} / \mathrm{s}$ ):

$$
V=\frac{Q}{A}
$$

Length measured along the axis (m):

```
L}=2\cdot\pi\cdotr\cdot\frac{\alpha}{360
```

Mass flow rate ( $\mathrm{kg} / \mathrm{s}$ ):
$G=Q \cdot \rho_{m}$

Fluid volume $\left(m^{3}\right)$ :

$$
\mathrm{Vol}=A \cdot L
$$

Fluid mass (kg):

$$
\text { Mas }=\mathrm{Vol} \cdot \rho_{m}
$$

Reynolds number:

$$
N_{\operatorname{Re}}=\frac{V \cdot d_{h}}{v}
$$

Relative roughness:

$$
R_{r}=\frac{\varepsilon}{d_{h}}
$$

Darcy friction factor:
$f=\frac{1}{\left[2 \cdot \log \left(\frac{\varepsilon}{3.7 \cdot d_{h}}+\frac{2.51}{N_{\mathrm{Re}} \cdot \sqrt{f}}\right)\right]^{2}}$

Colebrook-White equation ([1] equation 3.6)


Total pressure loss coefficient (based on mean velocity in bend):

$$
\mathrm{K}=f \cdot \alpha \cdot \frac{r}{d_{h}}+(0.10+2.4 \cdot f) \cdot \sin (\alpha / 2)+\frac{6.6 \cdot f \cdot(\sqrt{\sin (\alpha / 2)}+\sin (\alpha / 2))}{\left(\frac{r}{d_{h}}\right)^{\frac{4 \cdot \alpha}{\pi}}}
$$

Smooth bend with rectangular cross-section
Pressure loss coefficient ( $f=0.02$ )
Pipe Flow - Guide (2012) - Equation 15.1

0.02)


$$
\text { Pressure loss coefficient }(f=0.02)
$$

Pipe Flow - Guide (2012) - Equation 15.1
([1] equation 15.1 with $f=$
0.02)

Total pressure loss $(\mathrm{Pa})$ :
$\Delta P=K \cdot \frac{\rho_{m} \cdot V^{2}}{2}$

Total head loss of fluid (m):
$\Delta H=K \cdot \frac{V^{2}}{2 \cdot g}$

Hydraulic power loss (W):

$$
W h=\Delta P \cdot Q
$$

Straight length of equivalent pressure loss ( $m$ ):
$L_{\text {eq }}=K \cdot \frac{d_{h}}{f}$

Symbols, Definitions, SI Units:
w Rectangular cross-section width ( $m$ )
$h \quad$ Rectangular cross-section height ( $m$ )
$d_{h} \quad$ Bend hydraulic diameter ( $m$ )
A Cross-section area ( $\mathrm{m}^{2}$ )
Q Volume flow rate ( $\mathrm{m}^{3} / \mathrm{s}$ )
V Mean velocity ( $\mathrm{m} / \mathrm{s}$ )
$L \quad$ Length measured along the axis ( $m$ )
$r \quad$ Radius of curvature ( $m$ )
$\alpha \quad$ Curvature angle ( ${ }^{\circ}$ )
$G \quad$ Mass flow rate (kg/s)
Vol Fluid volume ( $\mathrm{m}^{3}$ )
Mas Fluid mass (kg)
Ne Reynolds number ()
$\mathrm{R}_{\mathrm{r}} \quad$ Relative roughness ()
$\varepsilon \quad$ Absolute roughness of walls (m)
f Darcy friction factor
K Total pressure loss coefficient (based on mean velocity in bend) ()
$\Delta \mathrm{P} \quad$ Total pressure loss ( Pa )
$\Delta H \quad$ Total head loss of fluid ( m )
Wh Hydraulic power loss (W)
Leq Straight length of equivalent pressure loss ( $m$ )
$\rho_{\mathrm{m}} \quad$ Fluid density $\left(\mathrm{kg} / \mathrm{m}^{3}\right.$ )
$v \quad$ Fluid kinematic viscosity ( $\mathrm{m}^{2} / \mathrm{s}$ )
$9 \quad$ Gravitational acceleration $\left(\mathrm{m} / \mathrm{s}^{2}\right)$

## Validity range:

- turbulent flow regime $\left(\mathrm{N}_{\mathrm{Re}} \geq 10^{4}\right)$
- stabilized flow upstream of the bend
- curvature angle between $0^{\circ}$ and $180^{\circ}$
- this formulation is for circular passages, but can be reasonably applied to square ducts or to rectangular ducts of low aspect ratio


## Example of application:

Fluid characteristics

Fluid: | Water @ $1 \mathrm{~atm}[\mathrm{HC}]$ |
| :--- |
|  |
|  |
| Ref.: IAPWS IF97 |



References:
[1] Pipe Flow: A Practical and Comprehensive Guide. Donald C. Rennels and Hobart M. Hudson. (2012)

## HydrauCalc

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