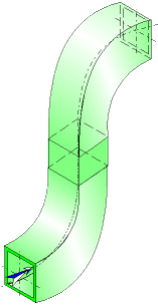




S-shaped Bends (with flow in one plane) Rectangular Cross-Section (IDELCHIK)



Model description:

This model of component calculates the head loss (pressure drop) of S-shaped bends (with flow in one plane) whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the first bend.

Model formulation:

Hydraulic diameter (m):

$$D_h = \frac{2 \cdot a_0 \cdot b_0}{a_0 + b_0} \quad ([1] \text{ diagram 6-1})$$

Cross-section area (m²):

$$F_0 = a_0 \cdot b_0$$

Total length measured along the axis (m):

$$l = 2 \cdot \left(2 \cdot \pi \cdot R_0 \cdot \frac{\delta}{360} \right) + l_{el}$$

Mean velocity (m/s):

$$w_0 = \frac{Q}{F_0}$$

Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

Fluid volume (m³):

$$V = F_0 \cdot l$$

Fluid mass (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$Re = \frac{w_0 \cdot D_h}{\nu}$$

Relative roughness:

$$\bar{\Delta} = \frac{\Delta}{D_h}$$

■ Case of relative radius of curvature lower than 3 ($R_0/b_0 < 3$) ([1] diagram 6-1)

Coefficient of effect of the roughness:

$$k_{\Delta} = f\left(\frac{R_0}{b_0}, Re, \bar{\Delta}\right) \quad ([1] \text{ diagram 6-1})$$

● $0.50 \leq R_0/b_0 \leq 0.55$

$\bar{\Delta}$	Re	
	$3 \cdot 10^3 - 4 \cdot 10^4$	$> 4 \cdot 10^4$
0	1.0	1.0
0 - 0.001	1.0	$1 + 0.5 \cdot 10^{-3} \cdot \bar{\Delta}$
> 0.001	1.0	1.5

● $R_0/b_0 > 0.55$

$\bar{\Delta}$	Re		
	$3 \cdot 10^3 - 4 \cdot 10^4$	$> 4 \cdot 10^4 - 2 \cdot 10^5$	$> 2 \cdot 10^5$
0	1.0	1.0	1.0
0 - 0.001	1.0	$\lambda_{\Delta} / \lambda_{sm}$	$1 + 10^{-3} \cdot \bar{\Delta}$
> 0.001	1.0	2.0	2.0

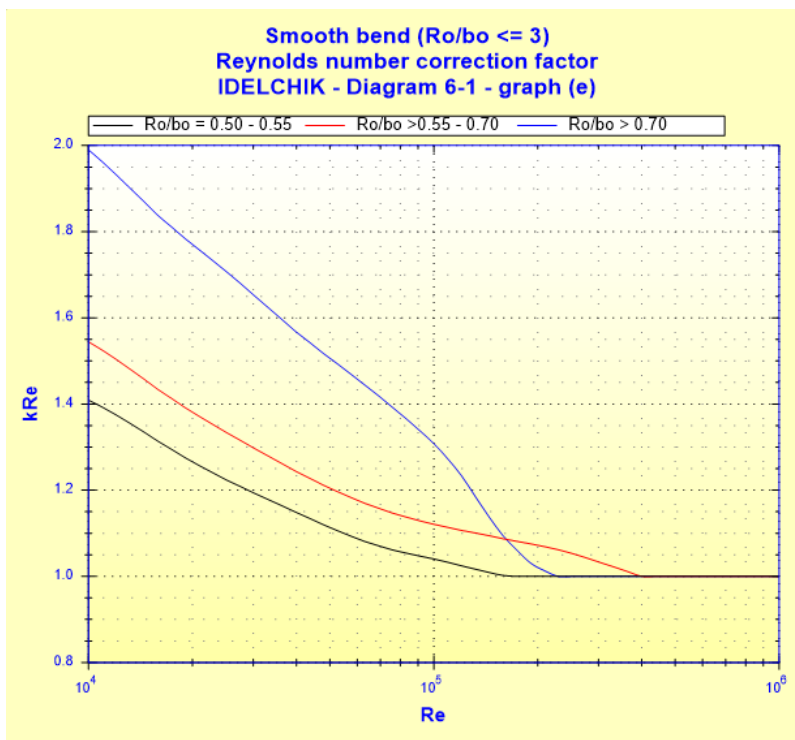
with:

λ_{sm} : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

λ_{Δ} : Darcy friction factor for rough pipe ($\bar{\Delta} = \Delta/D_h$) at Re

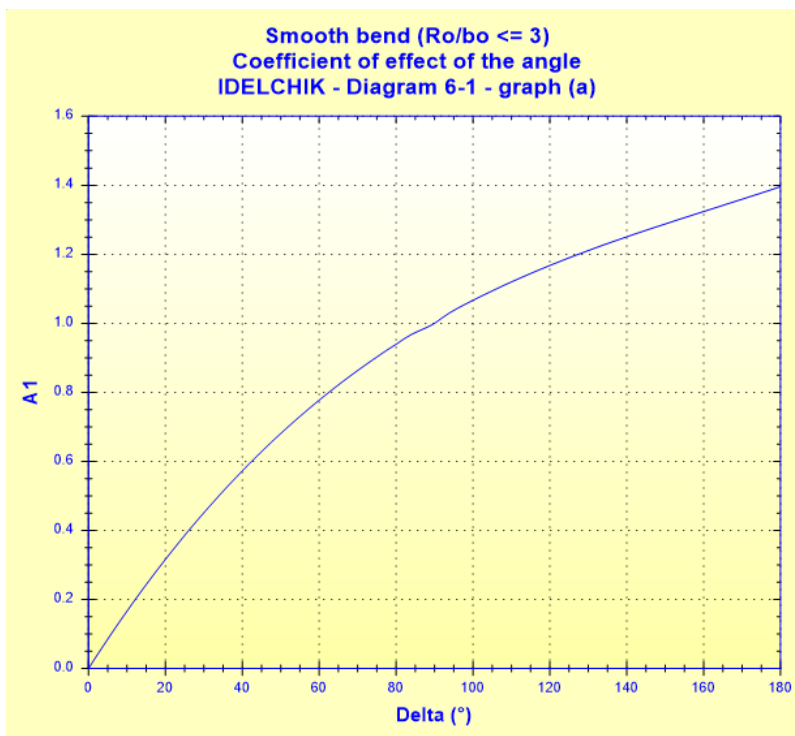
Coefficient of effect of the Reynolds number ($Re \geq 10^4$):

$$k_{Re} = f\left(Re, \frac{R_0}{b_0}\right) \quad ([1] \text{ diagram 6-1})$$



Coefficient of effect of the angle:

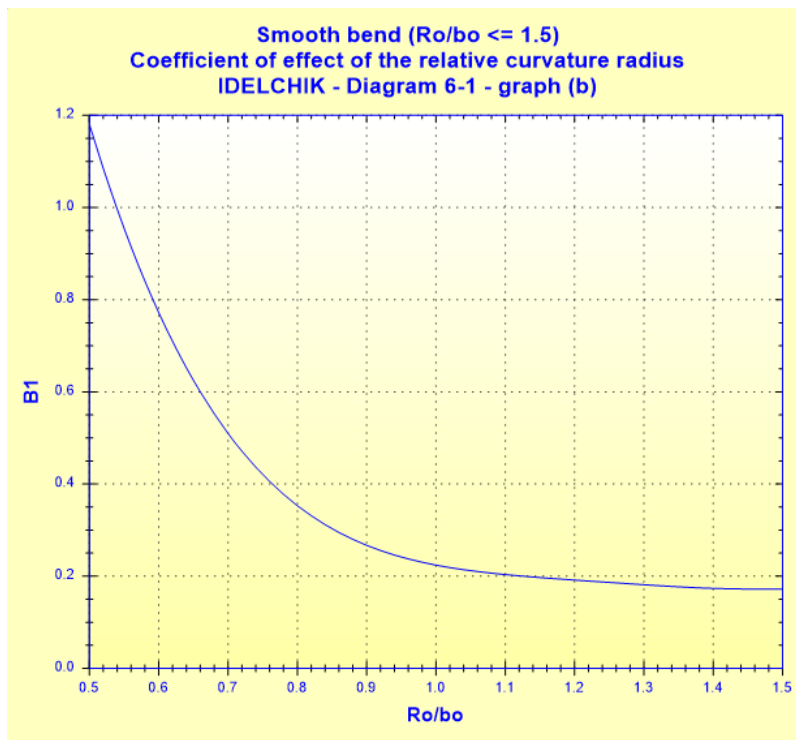
$$A1 = f(\delta) \quad ([1] \text{ diagram 6-1})$$



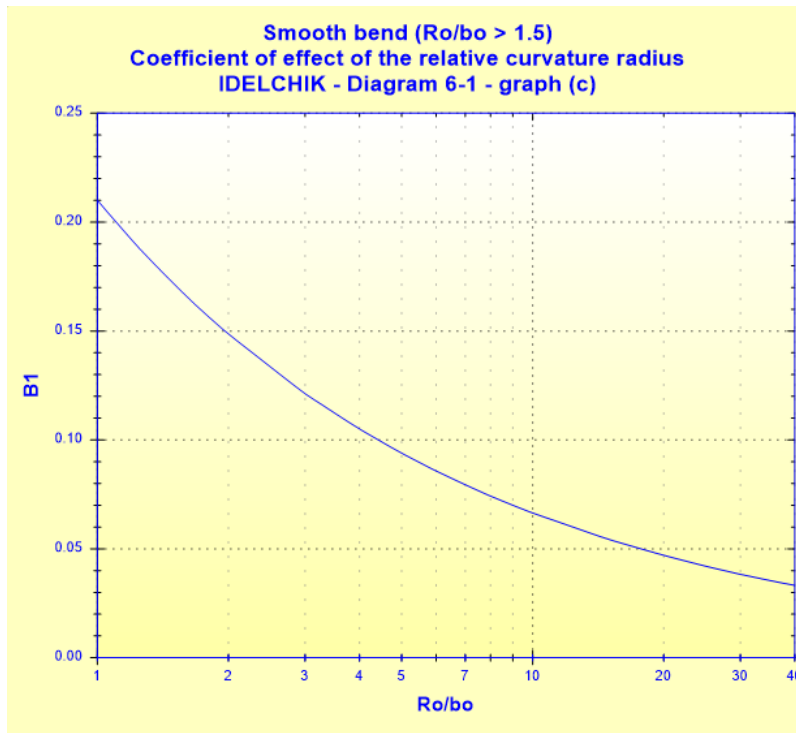
Coefficient of effect of the relative curvature radius:

$$B1 = f\left(\frac{R_0}{b_0}\right) \quad ([1] \text{ diagram 6-1})$$

- $0.5 \leq R_0/b_0 \leq 1.5$



● $R_0/b_0 > 1.5$



Coefficient of effect of the relative elongation of the cross section:

◆ $a_0 \geq b_0$

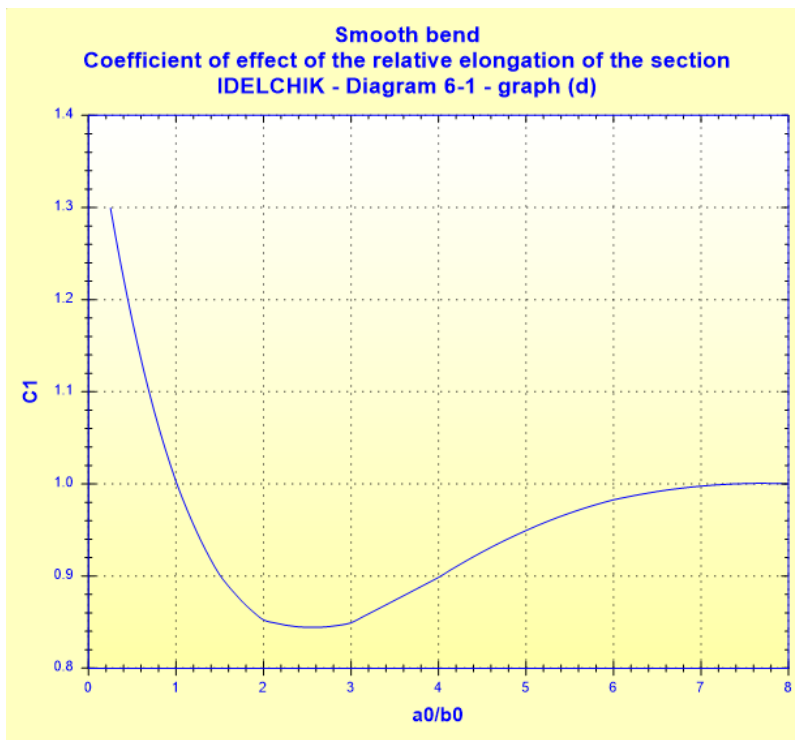
$$C_1 = f\left(\frac{a_0}{b_0}\right)$$

([1] diagram 6-1)

◆ $a_0 < b_0$

$$C_1 = f\left(\frac{b_0}{a_0}\right)$$

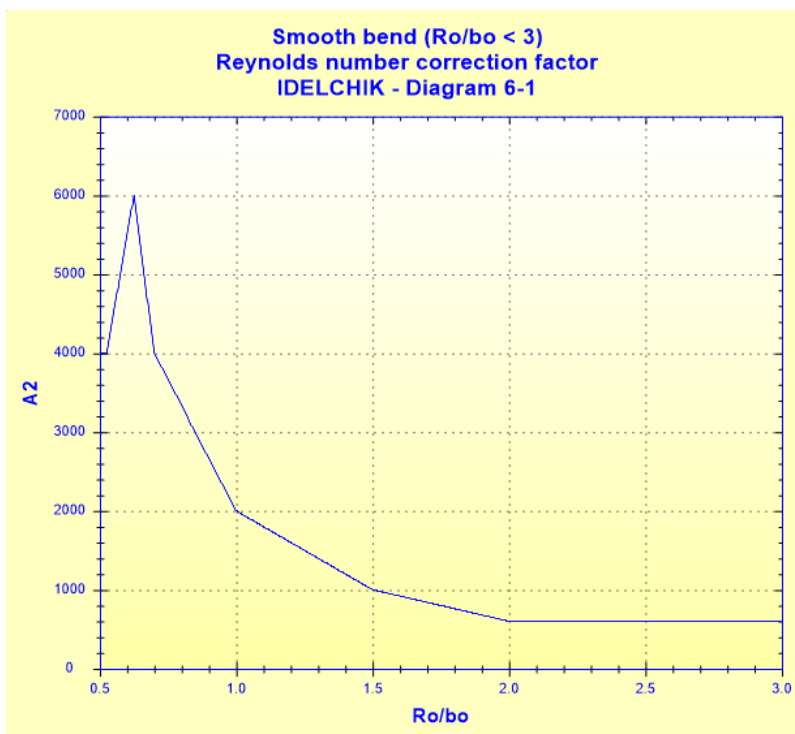
([1] diagram 6-1)



Reynolds number correction factor that depends on the relative curvature radius:

$$A2 = f\left(\frac{R_0}{b_0}\right) \quad ([1] \text{ diagram 6-1})$$

R_0/b_0	0.50 - 0.55	>0.55 - 0.70	>0.70 - 1.0	>1.0 - 2.0	>2.0
$A2 \times 10^{-3}$	4.0	6.0	4.0 - 2.0	1.0	0.6



Pressure loss coefficient (without friction):

- $Re \geq 10^4$

$$\zeta'_{loc} = k_{\Delta} \cdot k_{Re} \cdot A1 \cdot B1 \cdot C1 \quad ([1] \text{ diagram 6-1})$$

- $3 \cdot 10^3 < Re < 10^4$

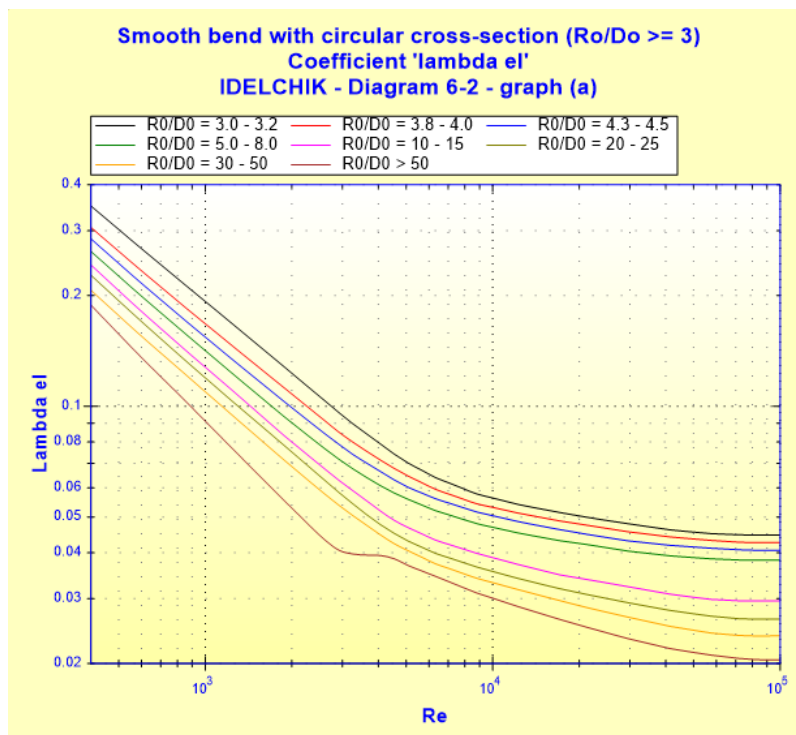
$$\zeta'_{loc} = \frac{A_2}{Re} + A_1 \cdot B_1 \cdot C_1 \quad ([1] \text{ diagram 6-1})$$

- Case of relative radius of curvature greater than or equal to 3 ($R_0/b_0 \geq 3$) ([1] diagram 6-2)

Total friction factor with smooth wall:

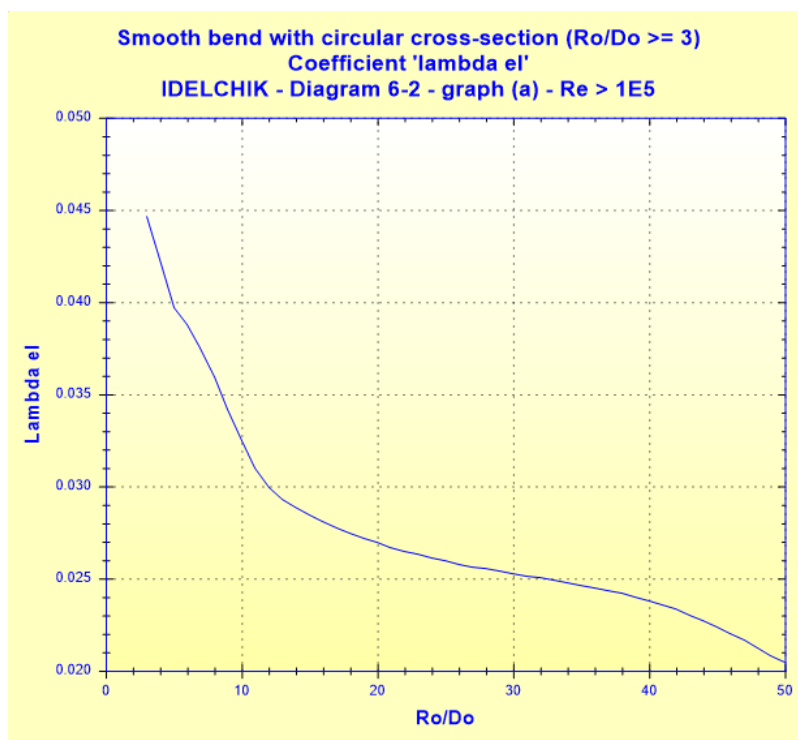
- $4 \cdot 10^2 \leq Re < 10^5$

$$\lambda_{el} = f\left(Re, \frac{R_0}{D_0}\right) \quad ([1] \text{ diagram 6-2})$$



- $Re \geq 10^5$

$$\lambda_{el} = f\left(\frac{R_0}{D_0}\right) \quad ([1] \text{ diagramme 6-2})$$



Estimation of the coefficient of local resistance

$$\zeta'_{loc} = (\lambda_{el} - \lambda_s) \cdot \frac{2 \cdot \pi \cdot R_0 \cdot \delta / 360}{D_h}$$

with:

λ_s : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

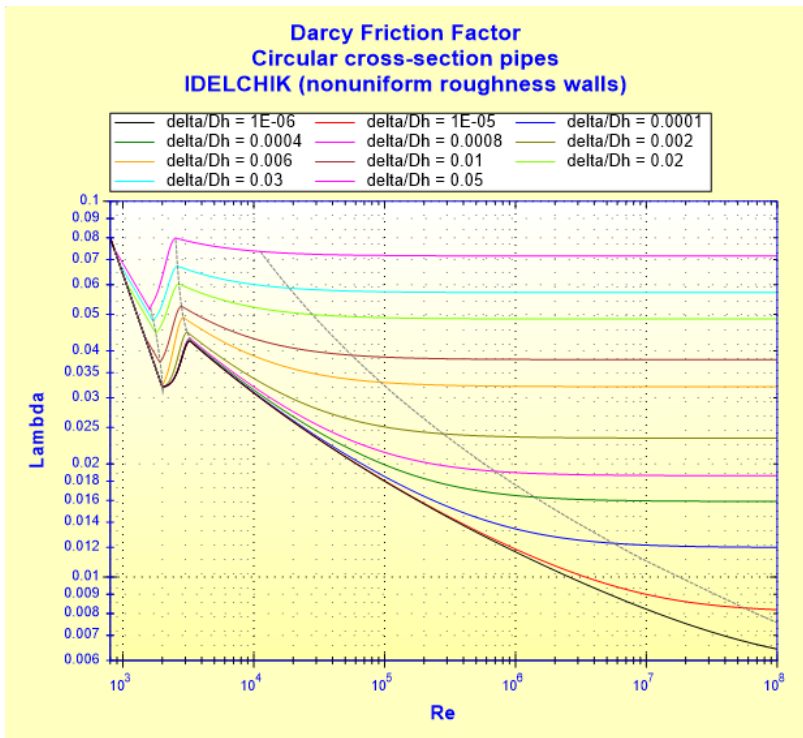
■ Case of the S-shaped Bends ([1] diagram 6-18)

Darcy friction factor:

See [Straight Pipe - Rectangular Cross-Section and Nonuniform Roughness Walls \(IDELCHIK\)](#)

■ Darcy friction factor for circular cross-section

$$\lambda_{circ} = f \left(Re, \frac{\Delta}{D_h} \right)$$



■ Correction for Darcy friction factor for noncircular cross-section

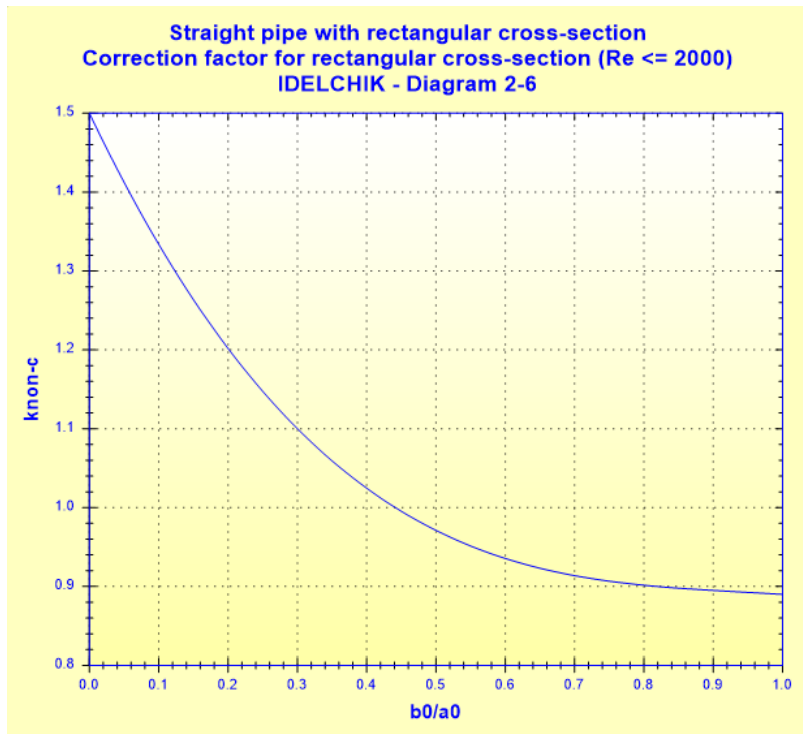
◆ $a_0 \geq b_0$

$$k_{non-c} = f(b_0/a_0) \quad ([1] \text{ diagram 2-6})$$

◆ $a_0 < b_0$

$$k_{non-c} = f(a_0/b_0) \quad ([1] \text{ diagram 2-6})$$

● laminar flow ($Re \leq 2000$):



● turbulent flow ($Re > 2000$):



■ Darcy friction factor for rectangular cross-section

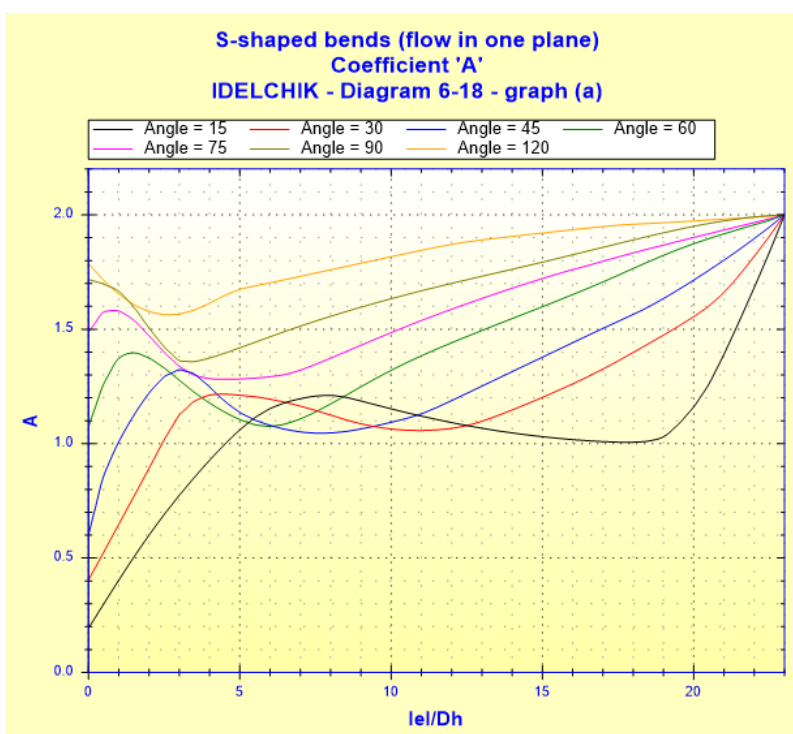
$$\lambda_{rect} = \lambda_{circ} \cdot k_{non-c} \quad ([1] \text{ diagram 2-6})$$

Pressure loss friction factor:

$$\zeta_{fr} = \lambda \cdot \left[2 \cdot \left(0.0175 \cdot \delta \cdot \frac{R_0}{D_h} \right) + \frac{l_{el}}{D_h} \right] \quad ([1] \text{ diagram 6-18})$$

Interaction correction factor:

$$A = f \left(\frac{l_{el}}{D_h}, \delta \right) \quad ([1] \text{ diagram 6-18 graph a})$$



Total pressure loss coefficient (based on the mean velocity in the bends):

$$\zeta = A \cdot \zeta'_{loc} + \zeta_{fr} \quad ([1] \text{ diagram 6-18})$$

Total pressure loss (Pa):

$$\Delta P = \zeta \cdot \frac{\rho \cdot w_0^2}{2} \quad ([1] \text{ diagram 6-18})$$

Total head loss of fluid (m):

$$\Delta H = \zeta \cdot \frac{w_0^2}{2 \cdot g}$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Straight length of equivalent pressure loss (m):

$$L_{eq} = \zeta \cdot \frac{D_h}{\lambda_{rect}}$$

Symbols, Definitions, SI Units:

a_0	Rectangular cross-section width (m)
b_0	Rectangular cross-section height (m)
D_h	Bend hydraulic diameter (m)
F_0	Cross-sectional area (m ²)
l	Total length measured along the axis (m)
R_0	Radius of curvature (m)
δ	Curvature angle of each bend (°)
Q	Volume flow rate (m ³ /s)
w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
V	Fluid volume (m ³)
M	Fluid mass (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\bar{\Delta}$	Relative roughness of walls ()
k_{Δ}	Coefficient that allows for the effect of the roughness ()
k_{Re}	Coefficient that allows for the effect of the Reynolds number ()
A_1	Coefficient that allows for the effect of the angle ()
B_1	Coefficient that allows for the effect of the relative curvature radius ()
C_1	Coefficient that allows for the effect of the relative elongation of the cross section ()
A_2	Reynolds number correction factor that depends on the relative curvature radius ()
ζ'_{loc}	Coefficient of local resistance ()
λ_{circ}	Darcy friction coefficient for circular cross-section ()

$k_{\text{non-c}}$	Correction for Darcy friction factor for noncircular cross-section ()
λ_{rect}	Darcy friction coefficient for rectangular cross-section ()
λ_{el}	Friction coefficient ()
ζ_{fr}	Pressure loss friction factor ()
A	Interaction correction factor ()
ζ	Total pressure loss coefficient (based on the mean velocity in the bend) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
L_{eq}	Straight length of equivalent pressure loss (m)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- stabilized flow upstream bend
- length of the straight section downstream: $\geq 10 D_h$
- relative radius of curvature: greater than or equal to 1 ($R_0/b_0 \geq 1$)
- curvature angle of one bend: between 0° and 180°

for ' δ ' angles less than 15° the pressure loss coefficient ' ζ ' is estimated by taking into account an interaction correction factor ' A ' corresponding to that of an angle of 15° .

for ' δ ' angles greater than 120° the pressure loss coefficient ' ζ ' is estimated by taking into account an interaction correction factor ' A ' corresponding to that of an angle of 120° .

- case of relative radius of curvature: lower than 3 ($R_0/b_0 < 3$)
 - flow regime: $Re \geq 3 \cdot 10^3$
- case of relative radius of curvature: greater than or equal to 3 ($R_0/b_0 \geq 3$)
 - flow regime: $500 \leq Re \leq 38 \cdot 10^3$

for Reynolds number ' Re ' lower than 500 or greater than $38 \cdot 10^3$, the coefficient ' λ_{el} ' is linearly extrapolated.

Example of application:

HydrauCalc 2023a - [S-shaped bends with rectangular cross-section (flow in one plane) - IDELCHIK (3rd Ed.)]

File Edit Preferences Calculation method Database Tools Help

Fluid characteristics
 Fluid : Water @ 1 atm [HC]
 Ref.: IAPWS IF97
 Temperature : T 20 °C
 Pressure : P 1.013 bar
 Density : ρ 998.2061 kg/m³
 Dynamic Viscosity : μ 0.00100159 N.s/m²
 Kinematic Viscosity : ν 1.00340E-06 m²/s
 Density Dyn. Visc. Kin. Visc.

Geometrical characteristics
 Help Info Calculate
 Pressure loss
 ΔP 0.001805047 bar
 ΔH 0.0184 m of fluid
 a0 0.1 m
 b0 0.05 m
 δ 90 °
 R0 0.175 m
 ℓel 0.1 m
 Δ 1.0E-05 m
 G 4.9910 kg/s
 Q 0.005 m³/s
 1.0 m/s (Turbulent)

Complementary results

Designation	Symbol	Value	Unit
Hydraulic diameter	Dh	0.06666667	m
Passage cross-section area	F0	0.005	m²
Sides ratio	b0/a0	0.5	
Relative radius of curvature	R0/b0	3.5	
Length between elbows / hydraulic diameter ratio	ℓel/Dh	1.5	
Relative roughness	Δ	0.00015	
Reynolds number	Re	66440.97	
Friction factor for rectangular cross-section	λ _{rect}	0.0208327	
<input checked="" type="checkbox"/> Friction factor (Diagram 6-2)	λ _{el}	0.04368597	
<input checked="" type="checkbox"/> Darcy friction factor (hydraulically smooth pipe)	λ _s	0.01962486	
<input checked="" type="checkbox"/> Interaction correction factor (Diagram 6-18)	A	1.59868	
Pressure loss coefficient (based on the mean bend velocity)	ζ	0.3616582	
Hydraulic power loss	Wh	0.9025233	W
Straight length of equivalent pressure loss	Leq	1.157341	m

References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik