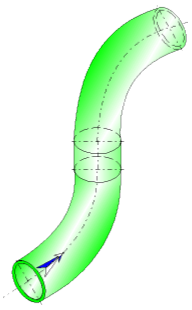
S-shaped Bends
(with flow in one plane)
Circular Cross-Section
(IDELCHIK)



Model description:

This model of component calculates the head loss (pressure drop) of S-shaped bends (with flow in one plane) whose cross-section is circular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the first bend.

Model formulation:

Hydraulic diameter (m):

$$D_h = D_0$$

Cross-section area (m²):

$$F_0 = \pi \cdot \frac{D_0^{\ 2}}{4}$$

Total length measured along the axis (m):

$$l = 2 \cdot \left(2 \cdot \pi \cdot R_0 \cdot \frac{\delta}{360} \right) + l_{el}$$

Mean velocity (m/s):

$$w_0 = \frac{Q}{F_0}$$

Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

Fluid volume (m³):

$$V = F_0 \cdot l$$

Fluid mass (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$Re = \frac{w_0 \cdot D_h}{\nu}$$

Relative roughness:

$$\overline{\Delta} = \frac{\Delta}{D_h}$$

■ Case of a single bend of relative radius of curvature lower than 3 ($R_0/D_0 < 3$) ([1] diagram 6-1)

Coefficient of effect of the roughness:

$$k_\Delta = f\left(\frac{R_0}{D_0}, Re, \overline{\Delta}\right)$$ ([1] diagram 6-1)

● $0.50 \le R_0/D_0 \le 0.55$

$\overline{\Delta}$	Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$
0	1.0	1.0
0 - 0.001	1.0	$1 + 0.5 \cdot 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	1.5

● $R_0/D_0 > 0.55$

$\overline{\Delta}$	Re		
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$ - $2 \cdot 10^5$	> $2 \cdot 10^5$
0	1.0	1.0	1.0
0 - 0.001	1.0	$\lambda_\Delta / \lambda_{sm}$	$1 + 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	2.0	2.0

with:

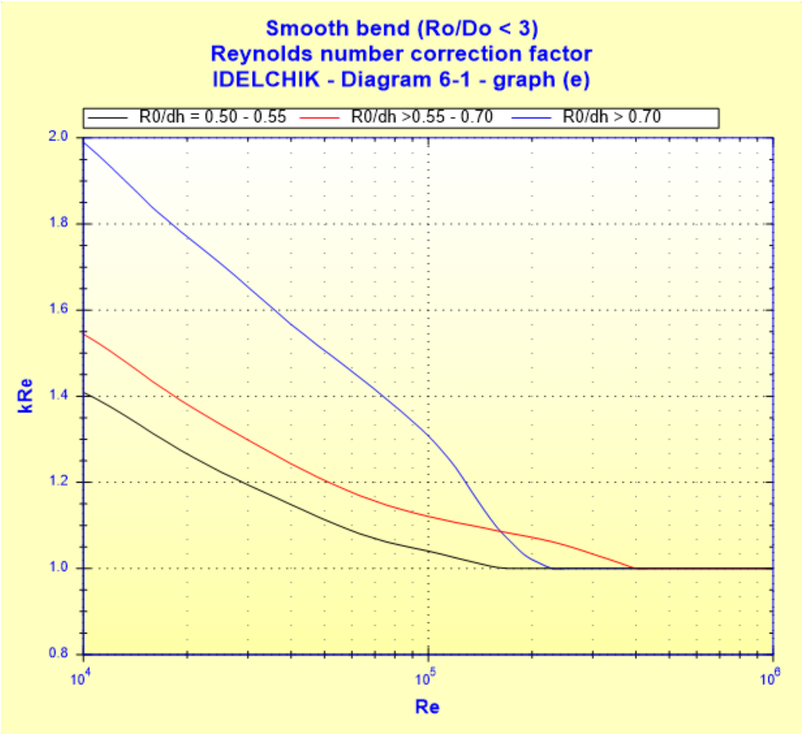
λ_{sm} : Darcy friction factor for hydraulically smooth pipe ($\overline{\Delta} = 0$) at Re

λ_Δ : Darcy friction factor for rough pipe ($\overline{\Delta} = \Delta/D_h$) at Re

Coefficient of effect of the Reynolds number ($Re \ge 10^4$):

$$k_{Re} = f\left(Re, \frac{R_0}{D_h}\right)$$

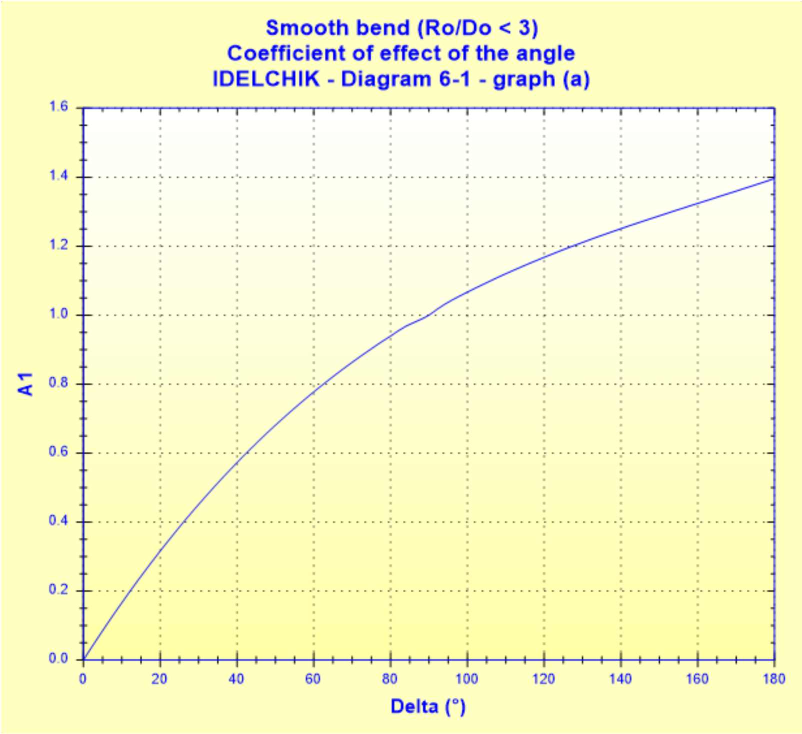
([1] diagram 6-1)



Coefficient of effect of the angle:

$$A1 = f(\delta)$$

([1] diagram 6-1)

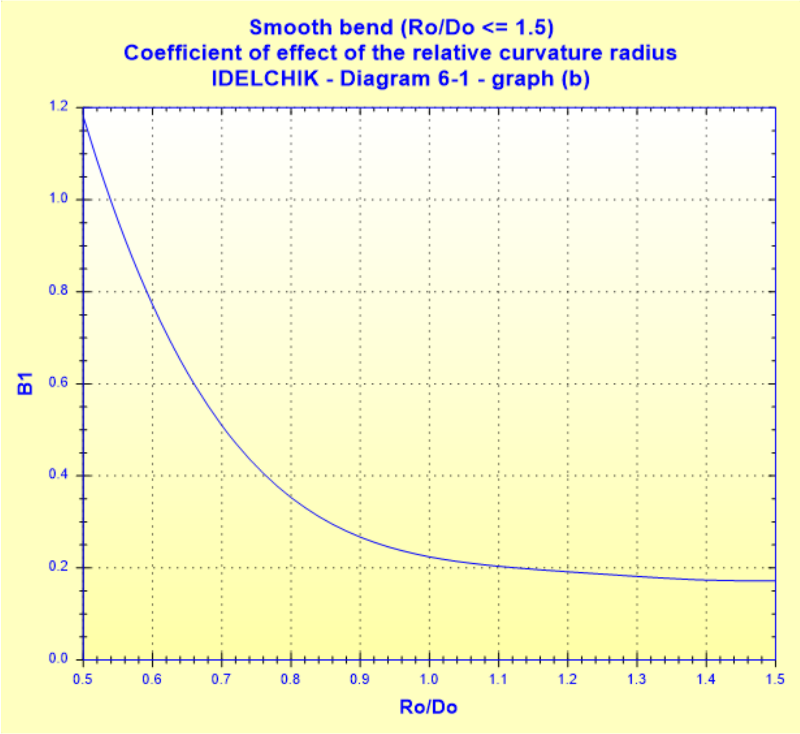


Coefficient of effect of the relative curvature radius:

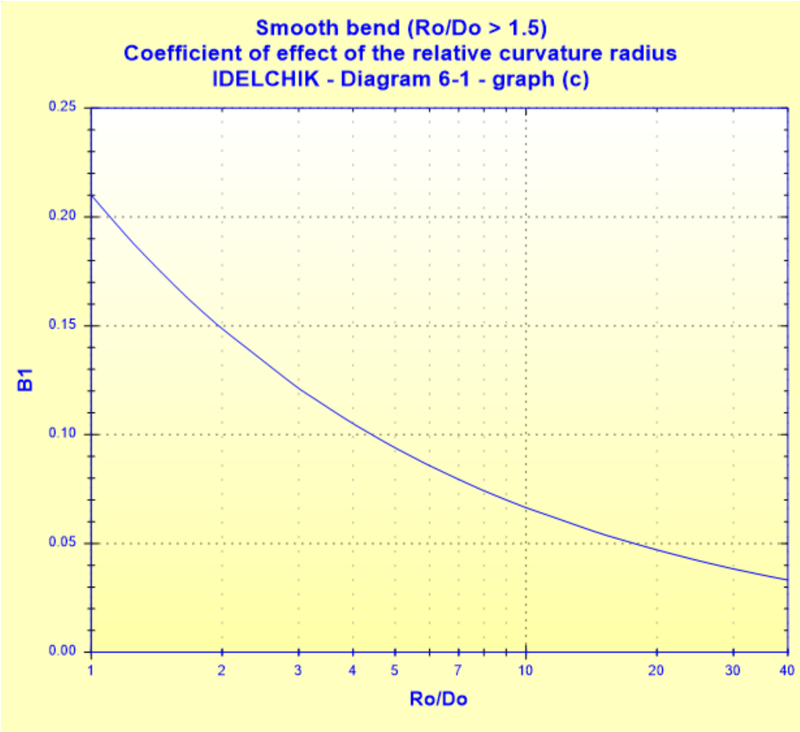
$$B1 = f\left(\frac{R_0}{D_h}\right)$$

([1] diagram 6-1)

- $0.5 \le R_0/D_0 \le 1.5$



- $R_0/D_0 > 1.5$



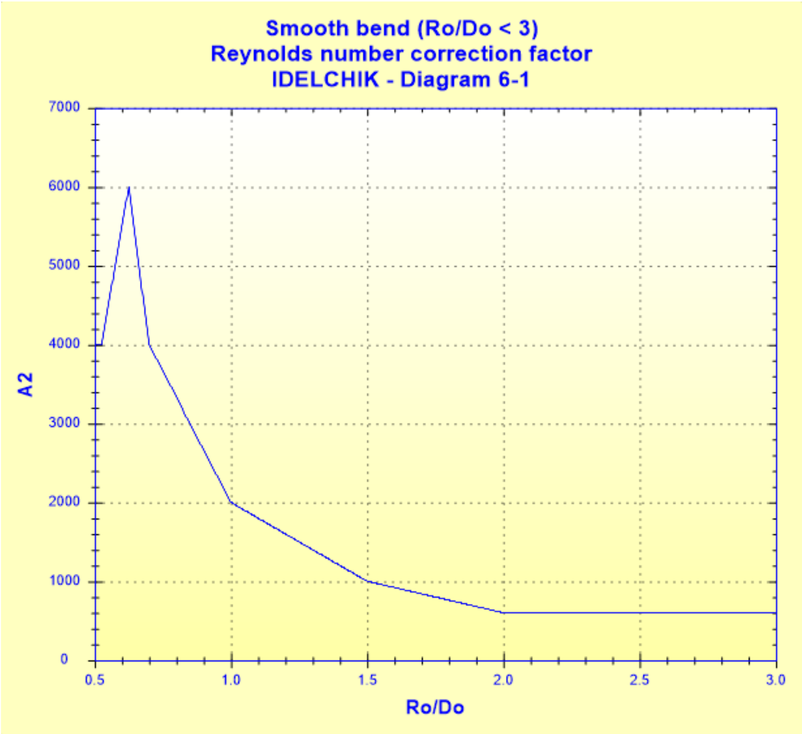
Coefficient of effect of the relative elongation of the cross section:

$C1 = 1$ ([1] diagram 6-1)

Reynolds number correction factor that depends on the relative curvature radius:

$A2 = f\left(\frac{R_0}{D_0}\right)$ ([1] diagram 6-1)

R_0/D_0	0.50 - 0.55	>0.55 - 0.70	>0.70 - 1.0	>1.0 - 2.0	>2.0
$A2 \times 10^{-3}$	4.0	6.0	4.0 - 2.0	1.0	0.6



Pressure loss coefficient (without friction):

- $Re \geq 10^4$

$$\zeta'_{loc} = k_\Delta \cdot k_{Re} \cdot A1 \cdot B1 \cdot C1$$ ([1] diagram 6-1)

- $3 \cdot 10^3 < Re < 10^4$

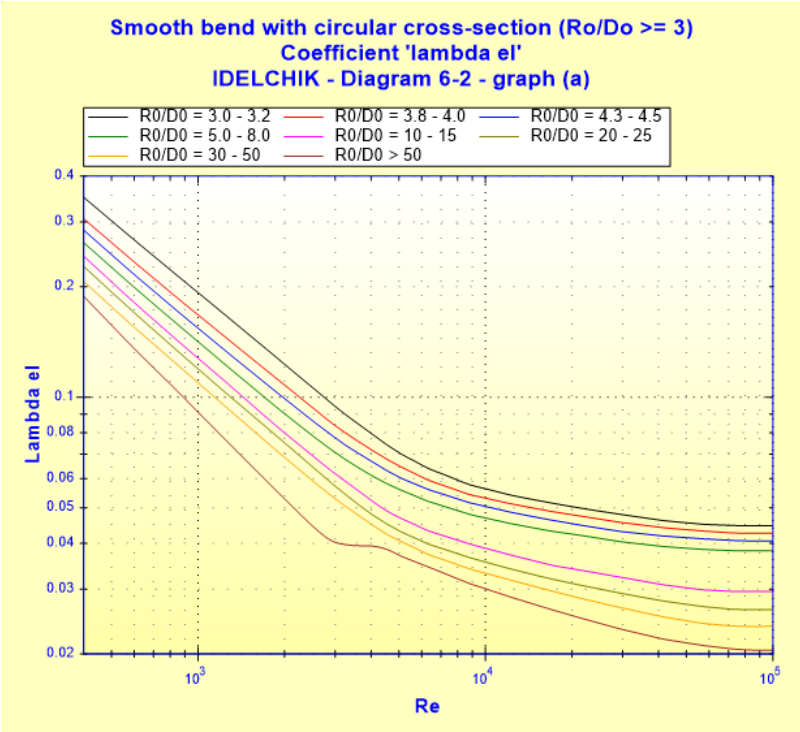
$$\zeta'_{loc} = \frac{A2}{Re} + A1 \cdot B1 \cdot C1$$ ([1] diagram 6-1)

■ Case of a single bend of relative radius of curvature greater than or equal to 3 ($R_0/D_0 \geq 3$) ([1] diagram 6-2)

Total friction factor with smooth wall:

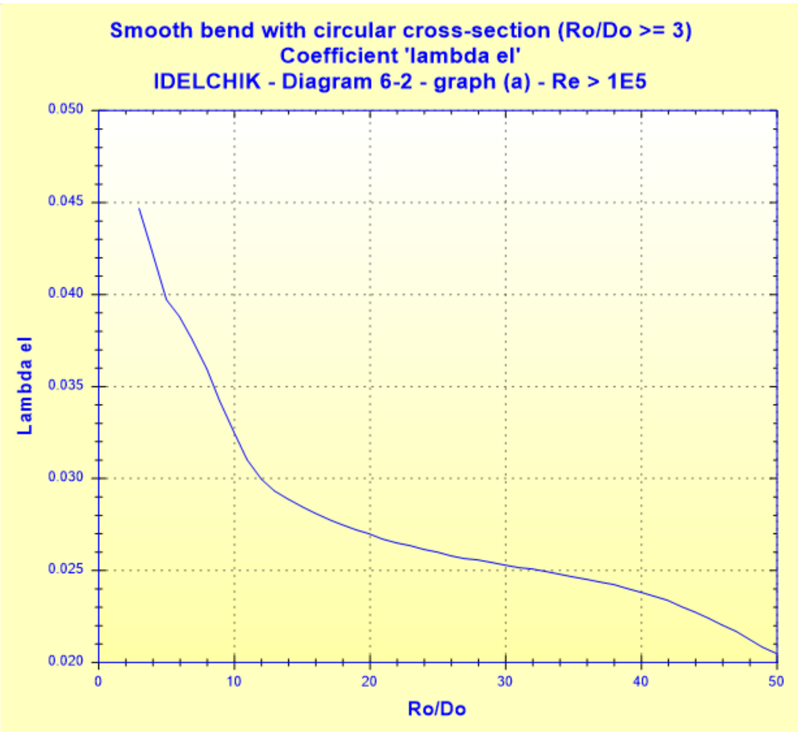
- $4 \cdot 10^2 \leq Re < 10^5$

$$\lambda_{el} = f\left(Re, \frac{R_0}{D_0}\right)$$ ([1] diagram 6-2)



- $Re \geq 10^5$

$$\lambda_{el} = f\left(\frac{R_0}{D_0}\right)$$ ([1] diagram 6-2)



Estimation of the coefficient of local resistance

$$\zeta'_{loc} = (\lambda_{el} - \lambda_s) \cdot \frac{2 \cdot \pi \cdot R_0 \cdot \delta / 360}{D_h}$$

with:

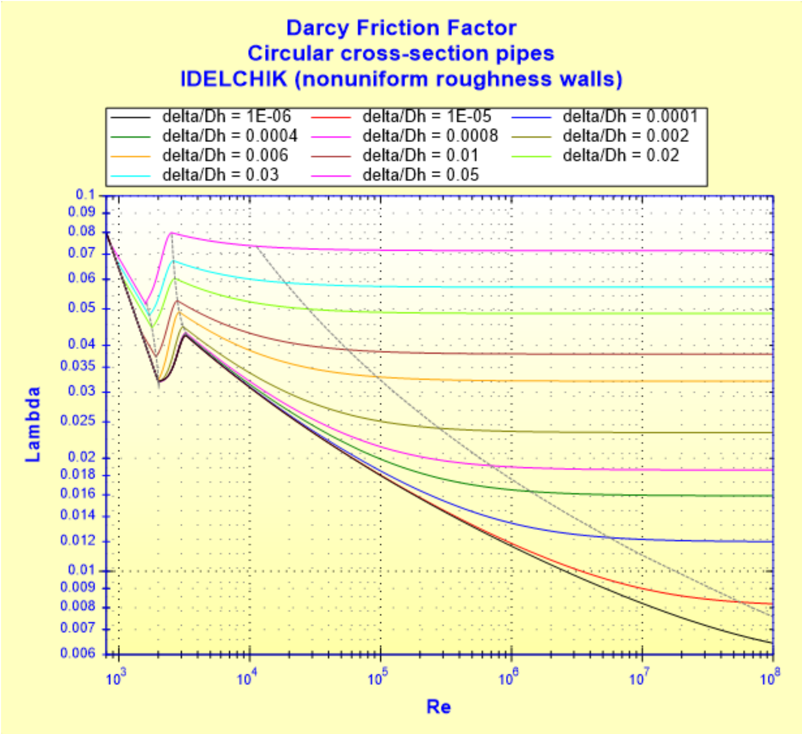
λ_s : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

■ Case of the S-shaped Bends ([1] diagram 6-18)

Darcy friction factor:

$$\lambda = f\left(\mathrm{Re}, \frac{\Delta}{D_h}\right)$$

See Straight Pipe - Circular Cross-Section and Nonuniform Roughness Walls (IDELCHIK)



Pressure loss friction factor:

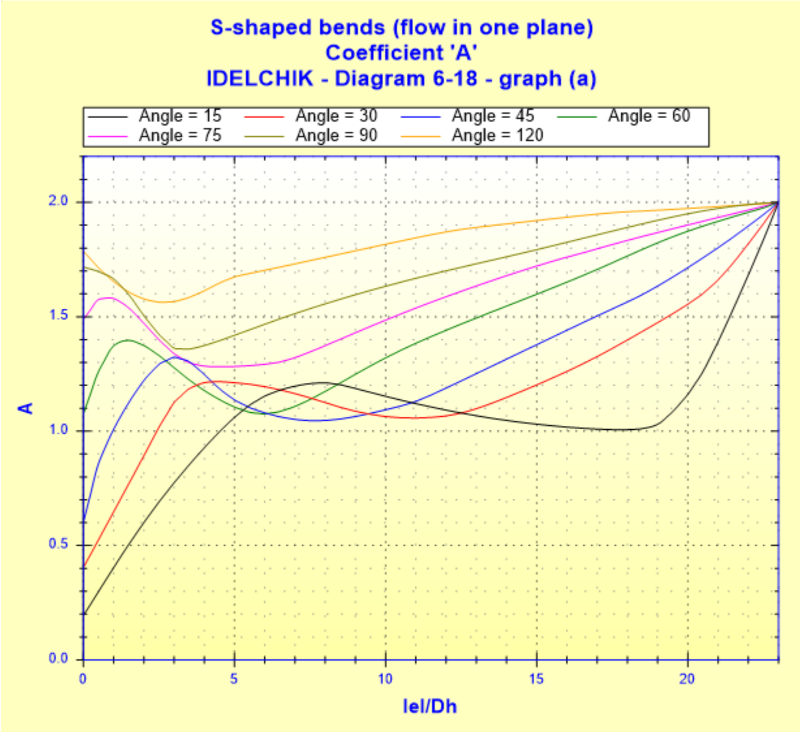
$$\zeta_{fr} = \lambda \cdot \left[2 \cdot \left(0.0175 \cdot \delta \cdot \frac{R_0}{D_h}\right) + \frac{l_{el}}{D_h}\right]$$

([1] diagram 6-18)

Interaction correction factor:

$$A = f\left(\frac{L_{el}}{D_h}, \delta\right)$$

([1] diagram 6-18 graph a)



Total pressure loss coefficient (based on the mean velocity in the bends):

$$\zeta = A \cdot \zeta'_{loc} + \zeta_{fr}$$

([1] diagram 6-18)

Straight length of equivalent pressure loss (m):

$$L_{eq} = \zeta \cdot \frac{D_0}{\lambda}$$

Total pressure loss (Pa):

$$\Delta P = \zeta \cdot \frac{\rho \cdot w_0^2}{2}$$

([1] diagram 6-18)

Total head loss of fluid (m):

$$\Delta H = \zeta \cdot \frac{w_0^2}{2 \cdot g}$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Symbols, Definitions, SI Units:

D_h	Bend hydraulic diameter (m)
D_0	Bend internal diameter (m)
F_0	Cross-sectional area (m^2)
l	Total length measured along the axis (m)
l_{el}	Straight length between bends (m)
R_0	Radius of curvature (m)
δ	Curvature angle of each bend (°)
Q	Volume flow rate (m^3/s)

w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
V	Fluid volume (m^3)
M	Fluid mass (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\overline{\Delta}$	Relative roughness of walls ()
k_Δ	Coefficient that allows for the effect of the roughness ()
k_{Re}	Coefficient that allows for the effect of the Reynolds number ()
A1	Coefficient that allows for the effect of the angle ()
B1	Coefficient that allows for the effect of the relative curvature radius ()
C1	Coefficient that allows for the effect of the relative elongation of the cross section ()
A2	Reynolds number correction factor that depends on the relative curvature radius ()
ζ'_{loc}	Coefficient of local resistance ()
λ_{el}	Total friction factor with smooth wall ()
λ	Darcy friction coefficient ()
ζ_{fr}	Pressure loss friction factor ()
A	Interaction correction factor ()
ζ	Total pressure loss coefficient (based on the mean velocity in the bend) ()
L_{eq}	Straight length of equivalent pressure loss (m)
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- stabilized flow upstream bend
- length of the straight section downstream: $\geq 10\ D_0$
- relative radius of curvature: greater than or equal to 1 ($R_0/D_0 \geq 1$)
- curvature angle of one bend: between 0° and 180°

  for 'δ' angles less than 15° the pressure loss coefficient 'ζ' is estimated by taking into account an interaction correction factor 'A' corresponding to that of an angle of 15°.

  for 'δ' angles greater than 120° the pressure loss coefficient 'ζ' is estimated by taking into account an interaction correction factor 'A' corresponding to that of an angle of 120°.

■ case of relative radius of curvature: lower than 3 ($R_0/D_0 < 3$)

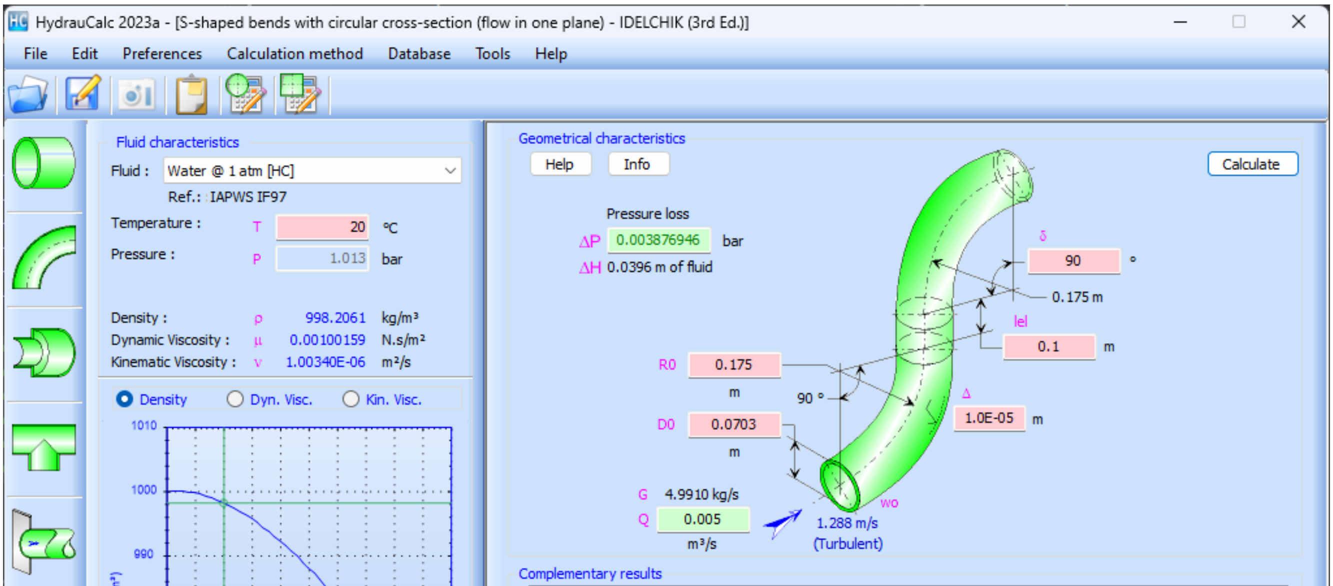
- flow regime: $Re \geq 3 \cdot 10^3$

■ case of relative radius of curvature: greater than or equal to 3 ($R_0/D_0 \geq 3$)

- flow regime: $Re \geq 400$

  for Reynolds number 'Re' lower than 400 the coefficient 'λ_{el}' is linearly extrapolated.

Example of application:



Designation	Symbol	Value	Unit
Hydraulic diameter	Dh	0.0703	m
Passage cross-section area	F0	0.003881508	m²
Relative radius of curvature	Ro/Do	2.489331	
Length between elbows / hydraulic diameter ratio	lel/Dh	1.422475	
Developed straight length from the axis	l	0.6497787	m
Relative roughness	$\bar{\Delta}$	0.0001422475	
Reynolds number	Re	90251	
Darcy Friction Factor	λ	0.01907611	
Coefficient of local resistance	ζ'loc	0.181306	
Coefficient of friction resistance	ζfr	0.1763193	
Interaction correction factor (Diagram 6-18)	A	1.609457	
Pressure loss coefficient (based on the mean bend velocity)	ζ	0.4681237	
Hydraulic power loss	Wh	1.938473	W
Straight length of equivalent pressure loss	Leq	1.725147	m

References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik