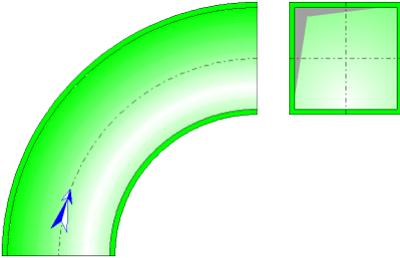




## Smooth Bend Rectangular Cross-Section (MILLER)



### Model description:

This model of component calculates the head loss (pressure drop) of a bend smoothly curved whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized upstream of the bend.

An option allows to take into account the effect of the straight length at the exit of the bend.

### Model formulation:

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Hydraulic diameter (m):

$$D = \frac{2 \cdot b \cdot W}{b + W} \quad ([1] \text{ equation 9.5})$$

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Cross-section area (m<sup>2</sup>):

$$A = b \cdot W$$

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Mean velocity (m/s):

$$U = \frac{Q}{A}$$

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Length measured along the axis (m):

$$L = 2 \cdot \pi \cdot r \cdot \frac{\theta_b}{360}$$

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Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

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Fluid volume (m<sup>3</sup>):

$$V = A \cdot L$$

Fluid mass (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$Re = \frac{U \cdot D}{\nu}$$

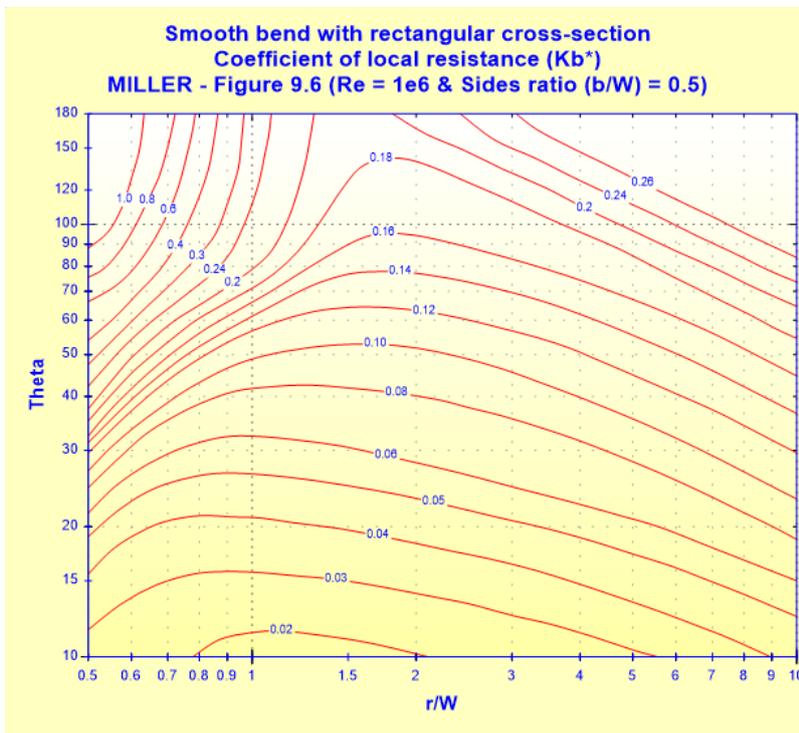
([1] equation 9.6)

Basic resistance coefficient:

$$K_b^* = f\left(\frac{r}{W}, \theta_b\right)$$

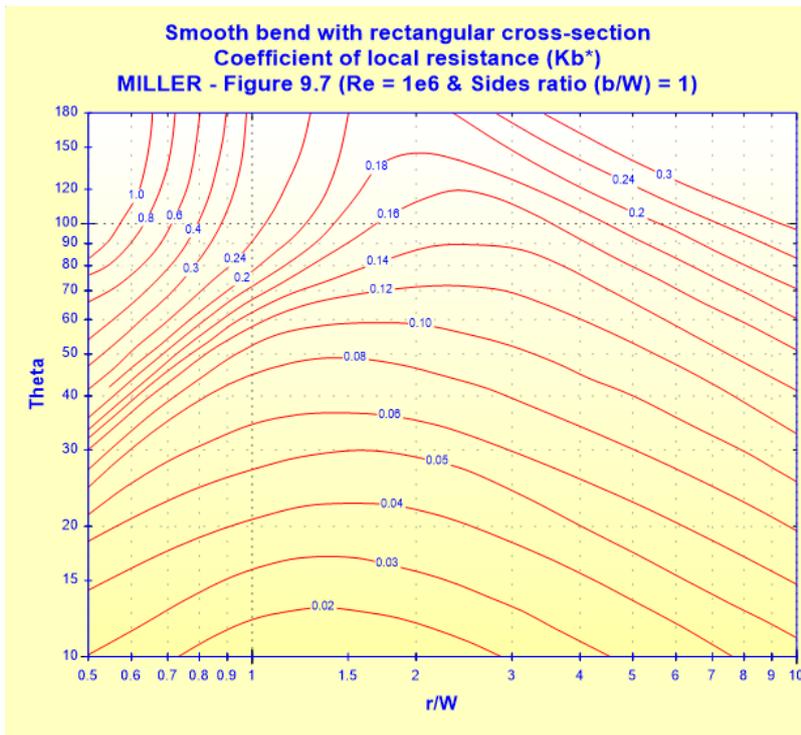
([1] figures 9.6 - 9.7 - 9.8)

■ Sides ratio  $b/W = 0.5$



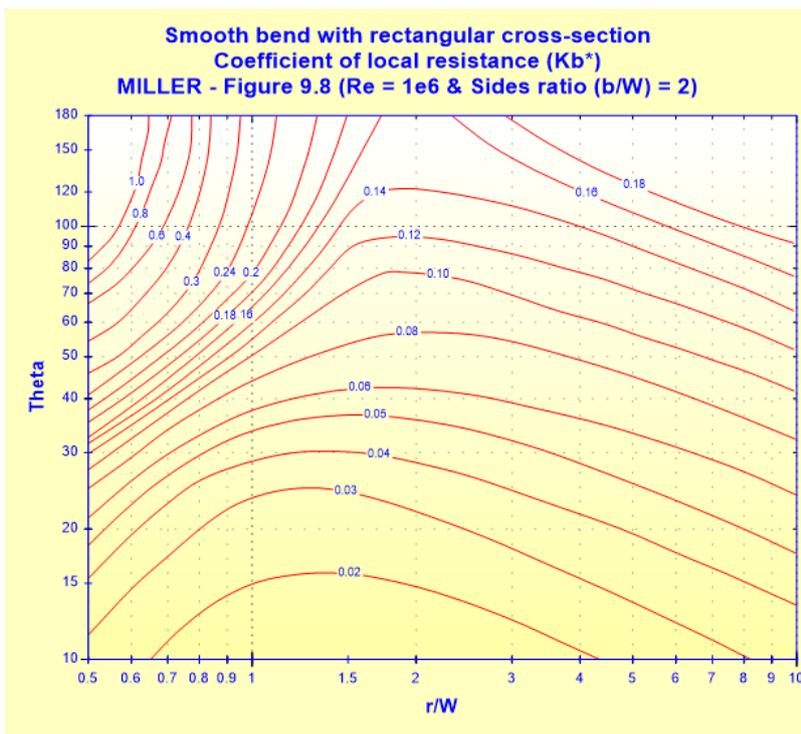
([1] figure 9.6)

■ Sides ratio  $b/W = 1$



([1] figure 9.7)

■ Sides ratio  $b/W = 2$

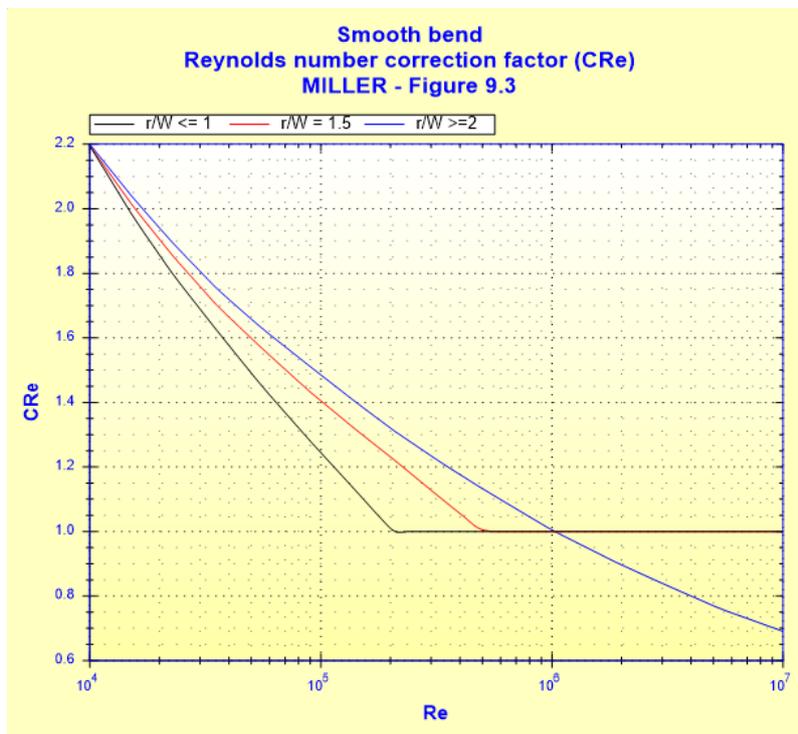


([1] figure 9.8)

For any sides ratio 'b/W' between 0.5 and 2, the coefficient  $K_b^*$  is obtained by curvilinear interpolation between the values of  $K_b^*$  calculated for aspect ratios of 0.5, 1 and 2.

Reynolds number correction factor:

$$C_{Re} = f\left(Re, \frac{r}{W}\right) \quad ([1] \text{ figure 9.3})$$



- $r/W \geq 1$

$$C_{Re} = f\left(Re, \frac{r}{D}\right) \quad ([1] \text{ figure 9.3})$$

- $r/W < 1$

- $r/W > 0.7$  or  $K_b^* < 0.4$

$$C_{Re} = f\left(Re, \frac{r}{D}\right) \quad ([1] \text{ figure 9.3 with } r/W=1)$$

- otherwise ( $r/W \leq 0.7$  and  $K_b^* \geq 0.4$ )

$$C_{Re} = \frac{K_b^*}{K_b^* - 0.2C'_{Re} + 0.2} \quad ([1] \text{ equation 9.2})$$

with:

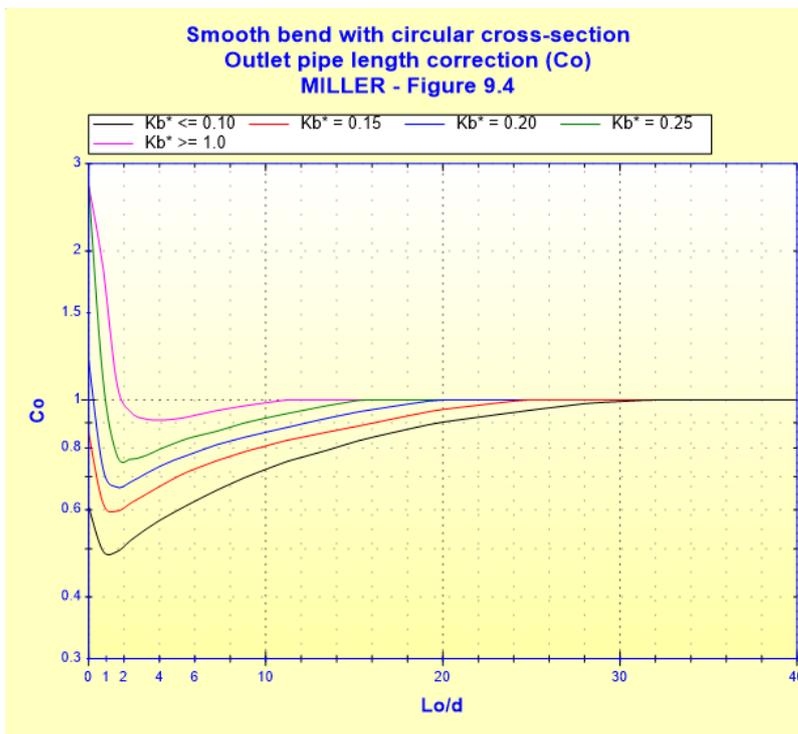
$$C'_{Re} = f\left(Re, \frac{r}{D}\right) \quad ([1] \text{ figure 9.3 with } r/W=1)$$

Outlet pipe length correction factor (optional):

- correction factor for circular cross-section

- $\theta_b < 100^\circ$

$$C_o = f\left(\frac{L_o}{D}, K_b^*\right) \quad ([1] \text{ figure 9.4})$$



- $\theta_b \geq 100^\circ$

$$\boxed{C_o = 1} \quad (\text{negligible effect})$$

■ correction factor for rectangular cross-section

- $b/W < 0.7$  and  $Lo/D > 1$

$$\boxed{C_{or} = 1 - \frac{1 - C_o}{2}}$$

- $b/W > 1$  and  $Lo/D < 1$

- ◆  $1.5 < r/W < 3$

$$\boxed{C_{or} = 2}$$

- ◆  $r/W \leq 1.5$  or  $r/W \geq 3$

$$\boxed{C_{or} = C_o}$$

- otherwise

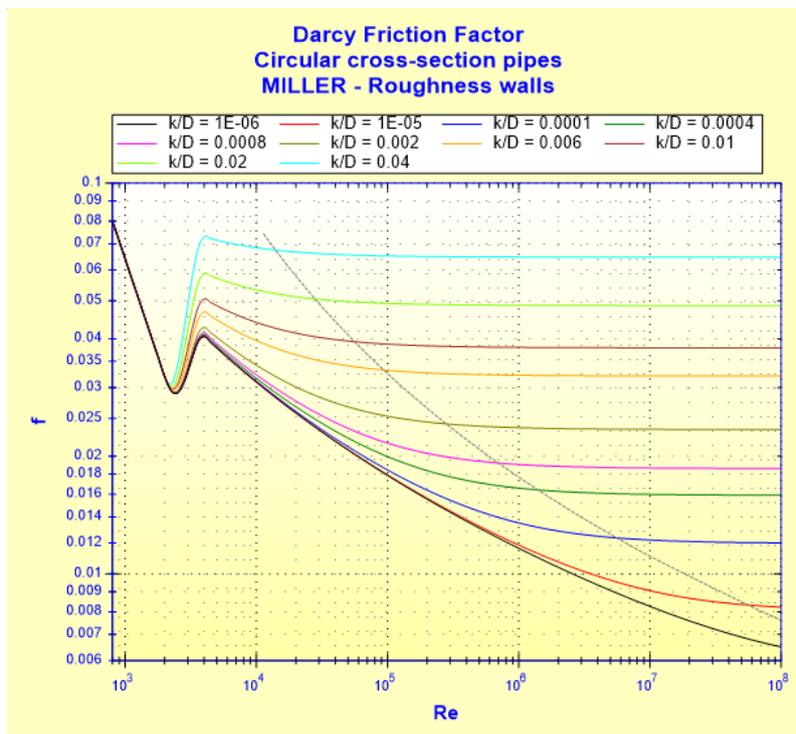
$$\boxed{C_{or} = C_o}$$

If this option is not activated, the factors  $C_o$  and  $C_{or}$  are equal to unity.

Darcy friction factor:

$$\boxed{f = f\left(\text{Re}, \frac{k}{D}\right)}$$

See [Straight Pipe - Rectangular Cross-Section and Roughness Walls \(MILLER\)](#)



Roughness correction factor:

$$C_f = \frac{f_{rough}}{f_{smooth}} \quad ([1] \text{ equation 9.3})$$

with:

$f_{rough}$ : Darcy friction factor for rough pipe at  $Re$

$f_{smooth}$ : Darcy friction factor for smooth pipe ( $k = 0$ ) at  $Re$

For  $Re > 10^6$ ,  $C_f$  is calculated from equation (9.3) for  $Re = 10^6$

Corrected loss coefficient:

$$K_b = K_b^* \cdot C_{Re} \cdot C_{or} \cdot C_f \quad ([1] \text{ equation 9.4})$$

Total pressure loss coefficient (based on the mean velocity in the bend)

$$K = K_b$$

Total pressure loss (Pa):

$$\Delta P = K \cdot \frac{\rho \cdot U^2}{2} \quad ([1] \text{ equation 8.1b})$$

Total head loss of fluid (m):

$$\Delta H = K \cdot \frac{U^2}{2 \cdot g} \quad ([1] \text{ equation 8.1a})$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Straight length of equivalent pressure loss (m):

$$L_{eq} = K \cdot \frac{D}{f_{rough}}$$

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**Symbols, Definitions, SI Units:**

W	Cross-section height (m)
b	Cross-section width (m)
D	Bend hydraulic diameter (m)
d	Bend internal diameter (m)
A	Cross-section area (m <sup>2</sup> )
Q	Volume flow rate (m <sup>3</sup> /s)
U	Mean velocity (m/s)
L	Length measured along the axis (m)
r	Radius of curvature (m)
$\theta_b$	Curvature angle (°)
G	Mass flow rate (kg/s)
V	Fluid volume (m <sup>3</sup> )
M	Fluid mass (kg)
Re	Reynolds number ( )
$K_b^*$	Basic loss coefficient ( )
$C_{Re}$	Reynolds number correction factor ( )
$C_o$	Outlet pipe length correction factor for circular cross-section ( )
$C_{or}$	Outlet pipe length correction factor for rectangular cross-section ( )
$L_0$	Length of the straight section downstream of the bend (m)
f	Darcy friction factor ( )
k	Absolute roughness of walls (m)
$C_f$	Roughness correction factor ( )
$K_b$	Corrected loss coefficient ( )
K	Total pressure loss coefficient (based on the mean velocity in the bend) ( )
$\Delta P$	Total pressure loss (Pa)
$\Delta H$	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
$L_{eq}$	Straight length of equivalent pressure loss (m)
$\rho$	Fluid density (kg/m <sup>3</sup> )
$\nu$	Fluid kinematic viscosity (m <sup>2</sup> /s)
g	Gravitational acceleration (m/s <sup>2</sup> )

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**Validity range:**

- turbulent flow regime ( $Re \geq 10^4$ )
- stabilized flow upstream bend
- curvature angle: between 10° and 180°
- relative radius of curvature 'r/W': between 0.5 and 10

- sides ratio 'b/W': between 0.5 and 2

note: for any sides ratio 'b/W' less than 0.5, the resistance coefficients  $K_b^*$  are obtained by linear extrapolation from the values of  $K_b^*$  calculated for sides ratios of 0.5 and 1.

for any sides ratio 'b/W' greater than 0.5 and 2, the resistance coefficients  $K_b^*$  are obtained by linear extrapolation from the values of  $K_b^*$  calculated for sides ratios of 0.5 and 1.

### Example of application:

The screenshot displays the HydraulCalc 2020a software interface for a smooth bend with a rectangular cross-section. The interface is divided into several sections:

- Fluid characteristics:**
  - Fluid: Water @ 1 atm [HC]
  - Ref.: IAPWS IF97
  - Temperature:  $T = 20$  °C
  - Pressure:  $P = 1.013$  bar
  - Density:  $\rho = 998.2061$  kg/m<sup>3</sup>
  - Dynamic Viscosity:  $\mu = 0.00100159$  N.s/m<sup>2</sup>
  - Kinematic Viscosity:  $\nu = 1.00340E-06$  m<sup>2</sup>/s
  - Selected property: Density
  - Graph: Density (kg/m<sup>3</sup>) vs Temperature (°C) showing a decreasing trend from 1000 at 10°C to approximately 950 at 100°C.
- Geometrical characteristics:**
  - Diagram of a 90-degree bend with a rectangular cross-section. Parameters shown include:
    - Width:  $W = 0.05$  m
    - Height:  $b = 0.1$  m
    - Radius of curvature:  $r = 0.175$  m
    - Relative radius of curvature:  $r/W = 3.5$
    - Basic coefficient:  $k = 1.0E-05$
    - Angle:  $\theta = 90$  °
    - Mass flow rate:  $m = 4.9910$  kg/s
    - Volume flow rate:  $Q = 0.005$  m<sup>3</sup>/s
    - Mean velocity:  $U = 1.0$  m/s (Turbulent)
  - Pressure loss:  $\Delta P = 0.001043333$  bar
  - Head loss:  $\Delta H = 0.0107$  m of fluid
  - Option: Outlet pipe length correction factor (unchecked)
  - Use outlet pipe length correction factor (unchecked)
  - Straight length:  $\geq 1.945852$  m
- Complementary results:**

Designation	Symbol	Value	Unit
Hydraulic diameter	$D$	0.06666667	m
Passage cross-section area	$A$	0.005	m <sup>2</sup>
Sides ratio	$b/W$	2	
Relative radius of curvature	$r/W$	3.5	
Developed straight length from the axis	$L$	0.2748893	m
Basic coefficient (Figures 9.6 9.7 9.8)	$K_b^*$	0.127294	
Reynolds number correction factor (Figure 9.3)	$CR_e$	1.585444	
Outlet tangent correction (Figure 9.4)	$C_o$	1	
Corrected coefficient $C_o$ for rectangular cross-section	$Cor$	1	
Relative roughness	$k/D$	0.00015	
Roughness correction	$C_f$	1.035796	
Reynolds number	$Re$	66440.97	
Corrected pressure loss coefficient	$K_b$	0.2090416	
Pressure loss coefficient (based on the mean bend velocity)	$K$	0.2090416	
Hydraulic power loss	$Wh$	0.5216665	W
Straight length of equivalent pressure loss	$Leq$	0.6902463	m

### References:

[1] Internal Flow System, Second Edition, D.S. Miller