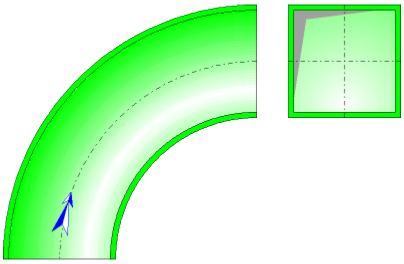
Smooth Bend
Rectangular Cross-Section
(IDELCHIK)



Model description:

This model of component calculates the head loss (pressure drop) of a bend smoothly curved whose cross-section is rectangular and constant. In addition, the flow is assumed fully developed and stabilized à the entrance bend.

Model formulation:

Hydraulic diameter (m):

$$\boxed{D_h = \frac{2 \cdot a_0 \cdot b_0}{a_0 + b_0}}$$

([1] diagram 6-1)

Cross-section area (m²):

$$\boxed{F_0 = a_0 \cdot b_0}$$

Length measured along the axis (m):

$$\boxed{l = 2 \cdot \pi \cdot R_0 \cdot \frac{\delta}{360}}$$

Mean velocity (m/s):

$$\boxed{w_0 = \frac{Q}{F_0}}$$

Mass flow rate (kg/s):

$$\boxed{G = Q \cdot \rho}$$

Fluid volume (m³):

$$\boxed{V = F_0 \cdot l}$$

Fluid mass (kg):

$$\boxed{M = V \cdot \rho}$$

Reynolds number:

$$\boxed{\mathrm{Re} = \frac{w_0 \cdot D_h}{\nu}}$$

Relative roughness:

$$\boxed{\overline{\Delta} = \frac{\Delta}{D_h}}$$

■ Case of relative radius of curvature lower than 3 ($R_0/b_0 < 3$) ([1] diagram 6-1)

Coefficient of effect of the roughness:

$$\boxed{k_\Delta = f\left(\frac{R_0}{b_0}, \mathrm{Re}, \overline{\Delta}\right)}$$ ([1] diagram 6-1)

● $0.50 \le R_0/b_0 \le 0.55$

$\overline{\Delta}$	Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$
0	1.0	1.0
0 - 0.001	1.0	$1 + 0.5 \cdot 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	1.5

● $R_0/b_0 > 0.55$

$\overline{\Delta}$		Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$ - $2 \cdot 10^5$	> $2 \cdot 10^5$
0	1.0	1.0	1.0
0 - 0.001	1.0	$\lambda_\Delta / \lambda_{sm}$	$1 + 10^3 \cdot \overline{\Delta}$
> 0.001	1.0	2.0	2.0

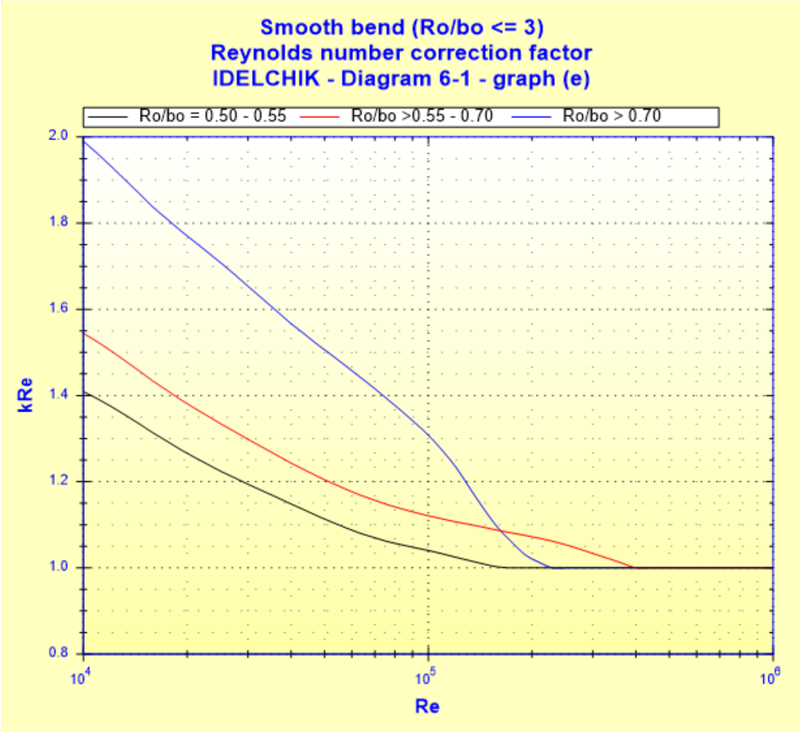
with:

λ_{sm} : Darcy friction factor for hydraulically smooth pipe ($\overline{\Delta} = 0$) at Re

λ_Δ : Darcy friction factor for rough pipe ($\overline{\Delta} = \Delta/D_h$) at Re

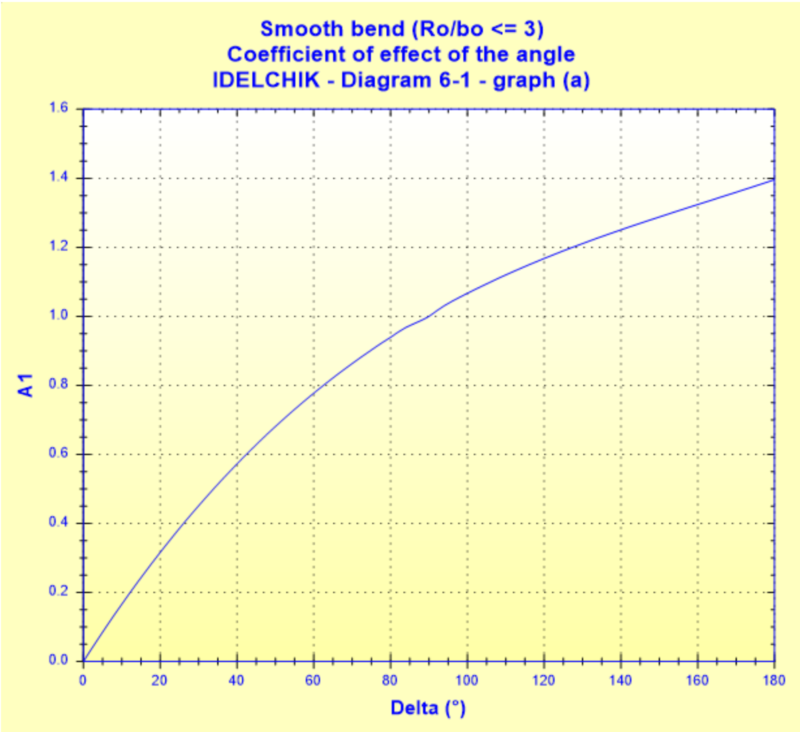
Coefficient of effect of the Reynolds number ($\mathrm{Re} \ge 10^4$):

$$\boxed{k_{\mathrm{Re}} = f\left(\mathrm{Re}, \frac{R_0}{b_0}\right)}$$ ([1] diagram 6-1)



Coefficient of effect of the angle:

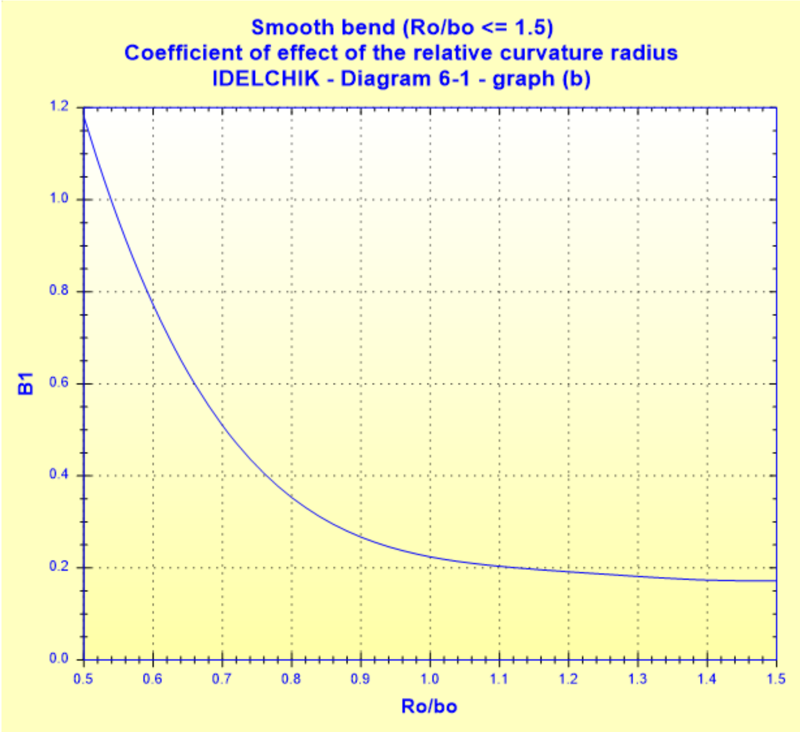
$\boxed{A1 = f(\delta)}$ ([1] diagram 6-1)



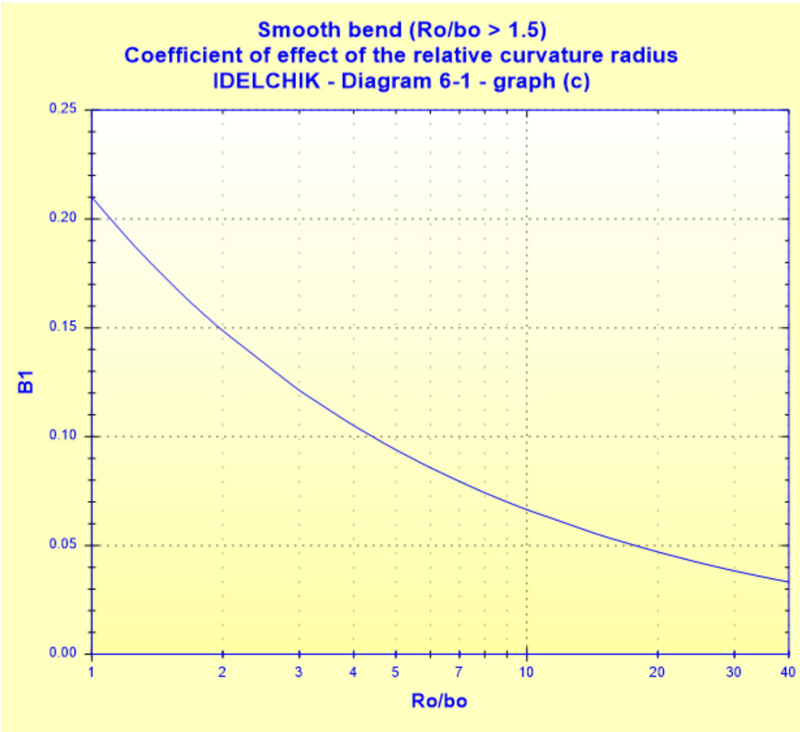
Coefficient of effect of the relative curvature radius:

$\boxed{B1 = f\left(\frac{R_0}{b_0}\right)}$ ([1] diagram 6-1)

- $0.5 \leq R_0/b_0 \leq 1.5$



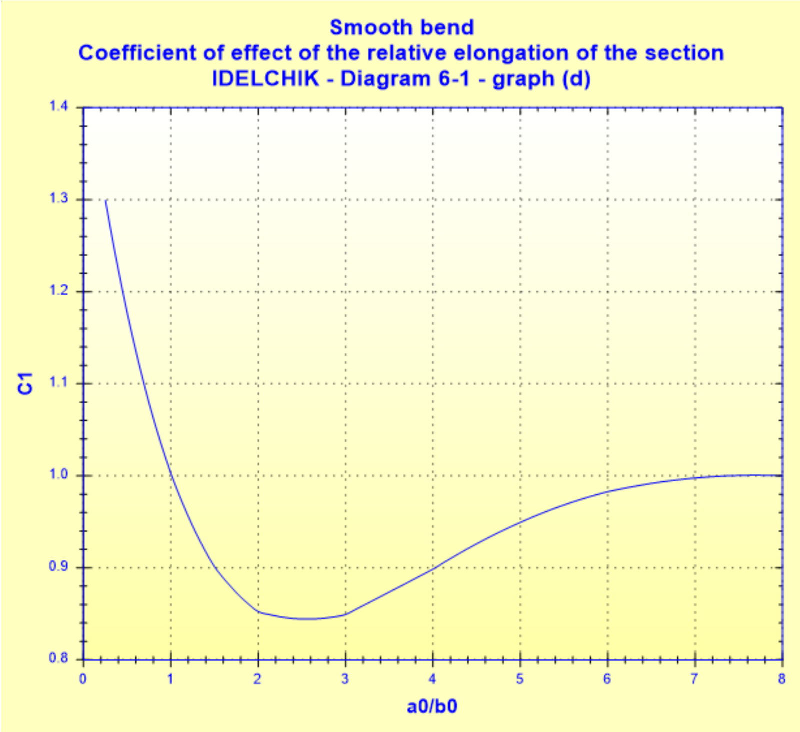
- $R_0/b_0 > 1.5$



Coefficient of effect of the relative elongation of the cross section:

$$\boxed{C1 = f\left(\frac{a_0}{b_0}\right)}$$

([1] diagram 6-1)



Coefficient of local resistance:

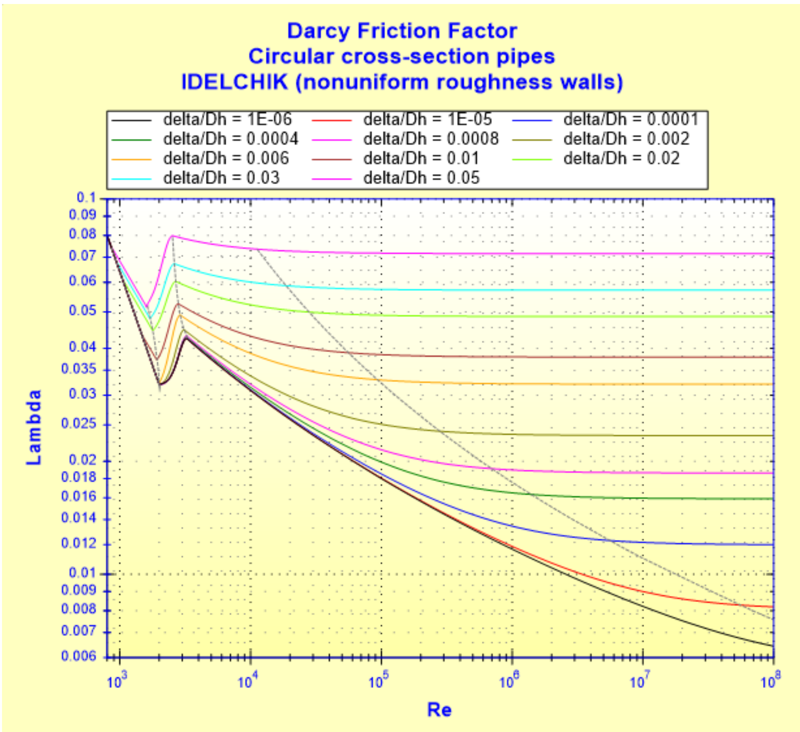
$$\boxed{\zeta_{loc} = A1 \cdot B1 \cdot C1}$$ ([1] diagram 6-1)

Darcy friction factor:

See [Straight Pipe - Rectangular Cross-Section and Nonuniform Roughness Walls (IDELCHIK)]

- Darcy friction factor for circular cross-section

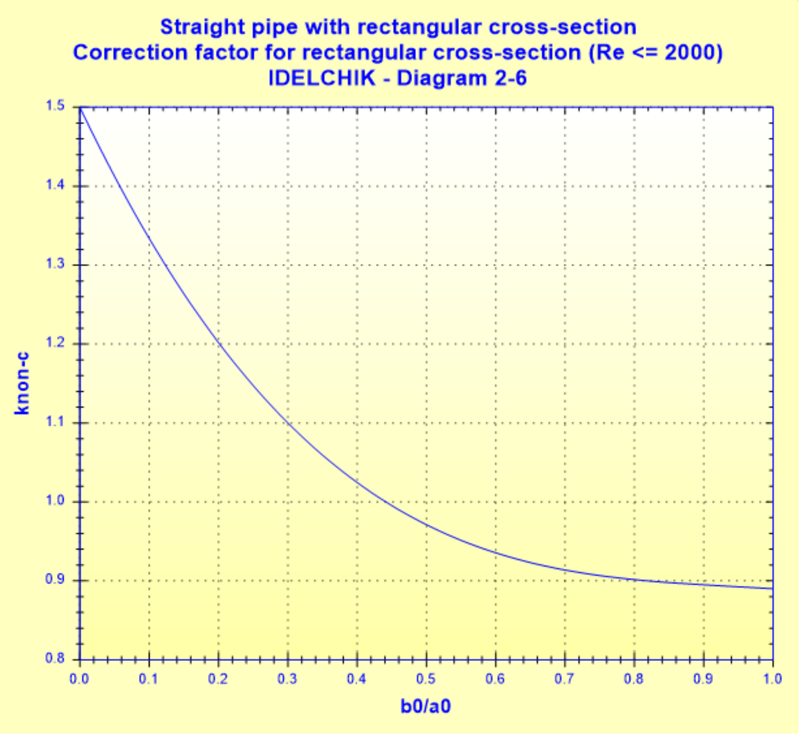
$$\boxed{\lambda_{circ} = f\left(\mathrm{Re}, \frac{\Delta}{D_h}\right)}$$



- Correction for Darcy friction factor for noncircular cross-section

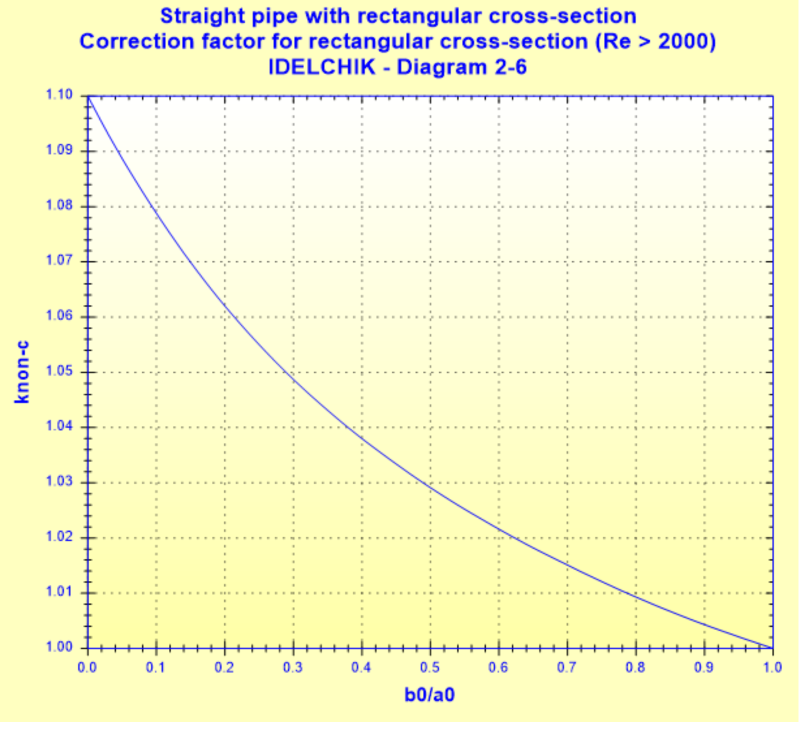
- laminar flow (Re $\leq$ 2000):

$\boxed{k_{non-c} = f(b_0/a_0)}$ ([1] diagram 2.6)



- turbulent flow (Re > 2000):

$\boxed{k_{non-c} = f(b_0/a_0)}$ ([1] diagram 2.6)



- Darcy friction factor for rectangular cross-section

$\boxed{\lambda_{rect} = \lambda_{circ} \cdot k_{non-c}}$ ([1] diagram 2.6)

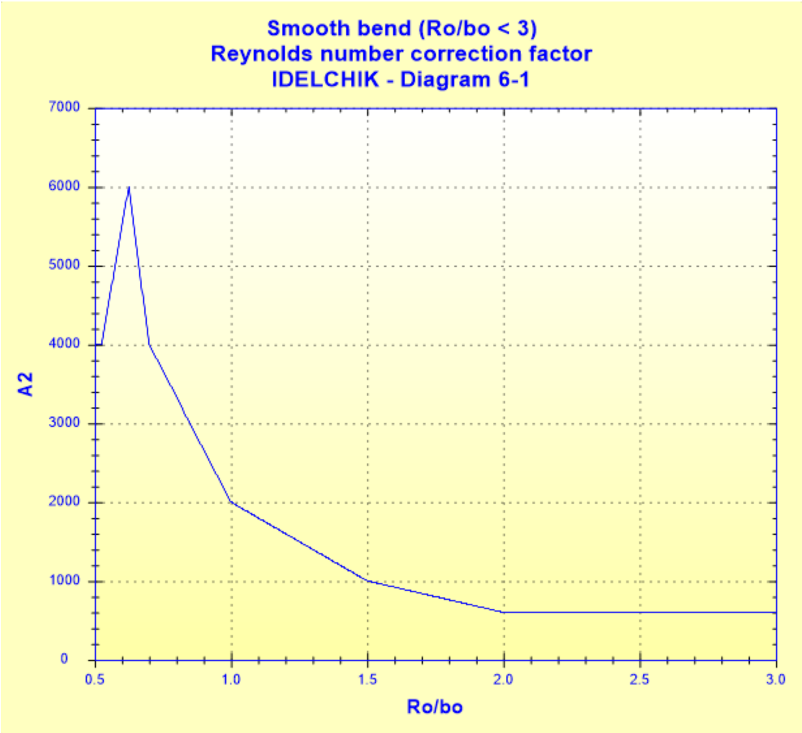
Pressure loss friction factor:

$$\zeta_{fr} = 0.0175 \cdot \delta \cdot \lambda_{rect} \cdot \frac{R_0}{D_h}$$ ([1] diagram 6-1)

Reynolds number correction factor that depends on the relative curvature radius:

$$A2 = f\left(\frac{R_0}{b_0}\right)$$ ([1] diagram 6.1)

R_0/b_0	0.50 - 0.55	>0.55 - 0.70	>0.70 - 1.0	>1.0 - 2.0	>2.0
$A2 \times 10^{-3}$	4.0	6.0	4.0 - 2.0	1.0	0.6



Total pressure loss coefficient:

- $Re \geq 10^4$

$$\zeta = k_\Delta \cdot k_{Re} \cdot \zeta_{loc} + \zeta_{fr}$$ ([1] diagram 6-1)

- $3 \cdot 10^3 < Re < 10^4$

$$\zeta = \frac{A2}{Re} + \zeta_{loc} + \zeta_{fr}$$ ([1] diagram 6-1)

Straight length of equivalent pressure loss (m):

$$L_{eq} = \zeta \cdot \frac{D_h}{\lambda_{rect}}$$

■ Case of relative radius of curvature greater than or equal to 3 ($R_0/b_0 \geq 3$) ([1] diagram 6-2)

Friction factor smooth wall:

- $0.5 \cdot 10^3 < Re < 6 \cdot 10^3$ (laminar regime)

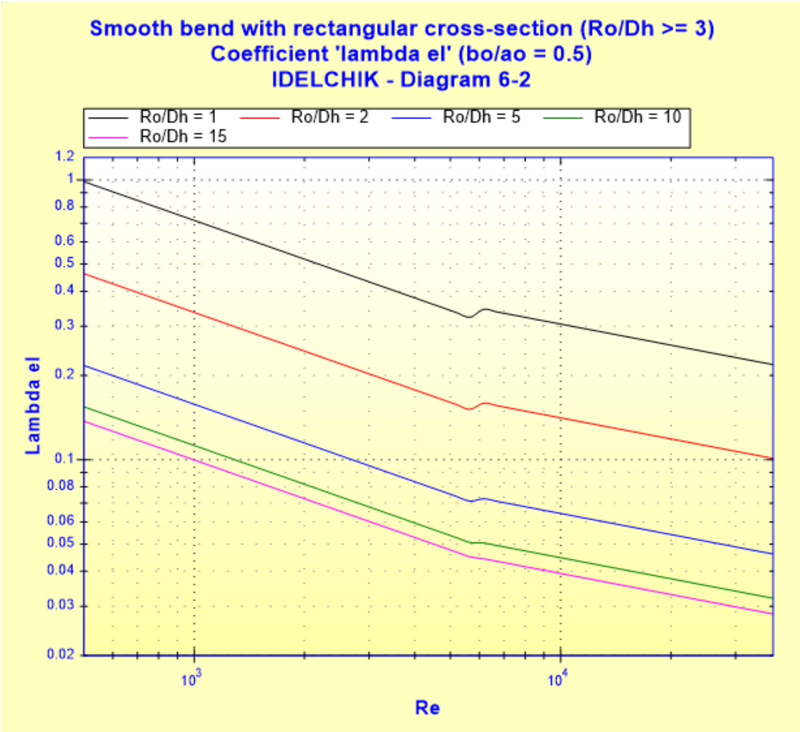
$$\boxed{A_{lam} = 1.97 + 49.1 \cdot \left(\frac{D_h}{2 \cdot R_0}\right)^{1.32} \cdot \left(\frac{b_0}{a_0}\right)^{0.37}}$$ ([1] diagram 6-2)

$$\boxed{\lambda_{el} = A_{lam} \cdot Re^{-0.46}}$$ ([1] diagram 6-2)

- $7 \cdot 10^3 < Re < 38 \cdot 10^3$ (turbulent regime)

$$\boxed{A_{turb} = 0.316 + 8.65 \cdot \left(\frac{D_h}{2 \cdot R_0}\right)^{1.32} \cdot \left(\frac{b_0}{a_0}\right)^{0.34}}$$ ([1] diagram 6-2)

$$\boxed{\lambda_{el} = A_{turb} \cdot Re^{-0.25}}$$ ([1] diagram 6.2)



([1] diagram 6-2 with $b_0/a_0 = 0.5$)

Roughness correction factor:

$$\boxed{C_f = \frac{\lambda_r}{\lambda_s}}$$ ([2] equation 9-3)

with:

λ_r : Darcy friction factor for rough pipe ($\bar{\Delta} = \Delta/D_h$) at Re
λ_s : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

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Pressure loss coefficient (based on the mean velocity in the bend):

$$\boxed{\zeta = 0.0175 \cdot \delta \cdot \lambda_{el} \cdot C_f \cdot \frac{R_0}{D_h}}$$ ([1] diagram 6-2 & [2] equation 9.3)

Straight length of equivalent pressure loss (m):

$$\boxed{L_{eq} = \zeta \cdot \frac{D_h}{\lambda}}$$

Total pressure loss (Pa):

$$\boxed{\Delta P = \zeta \cdot \frac{\rho \cdot w_0^{\;2}}{2}}$$

([1] diagram 6-1 – 6-2)

Total head loss of fluid (m):

$$\boxed{\Delta H = \zeta \cdot \frac{w_0^{\;2}}{2 \cdot g}}$$

Hydraulic power loss (W):

$$\boxed{Wh = \Delta P \cdot Q}$$

Symbols, Definitions, SI Units:

a_0	Rectangular cross-section width (m)
b_0	Rectangular cross-section height (m)
D_h	Bend hydraulic diameter (m)
F_0	Cross-sectional area (m²)
l	Length measured along the axis (m)
R_0	Radius of curvature (m)
δ	Curvature angle (°)
Q	Volume flow rate (m³/s)
w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
V	Fluid volume (m³)
M	Fluid mass (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\overline{\Delta}$	Relative roughness of walls ()
k_Δ	Coefficient that allows for the effect of the roughness
k_{Re}	Coefficient that allows for the effect of the Reynolds number
A1	Coefficient that allows for the effect of the angle
B1	Coefficient that allows for the effect of the relative curvature radius
C1	Coefficient that allows for the effect of the relative elongation of the cross section
ζ_{loc}	Coefficient of local resistance ()
λ_{circ}	Darcy friction coefficient for circular cross-section ()
λ_{rect}	Darcy friction coefficient for rectangular cross-section ()
k_{non-c}	Correction for Darcy friction factor for noncircular cross-section ()
ζ_{fr}	Pressure loss friction factor ()
ζ	Total pressure loss coefficient (based on the mean velocity in the bend) ()

L_{eq}	Straight length of equivalent pressure loss (m)
λ_{el}	Friction coefficient ()
A_{lam}	Laminar coefficient ()
A_{tur}	Turbulent coefficient ()
C_f	Roughness correction factor ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- stabilized flow upstream bend
- length of the straight section downstream: $\geq 10\ D_h$
- curvature angle: 0 to 180°

■ case of relative radius of curvature lower than 3 ($R_0/b_0 < 3$)

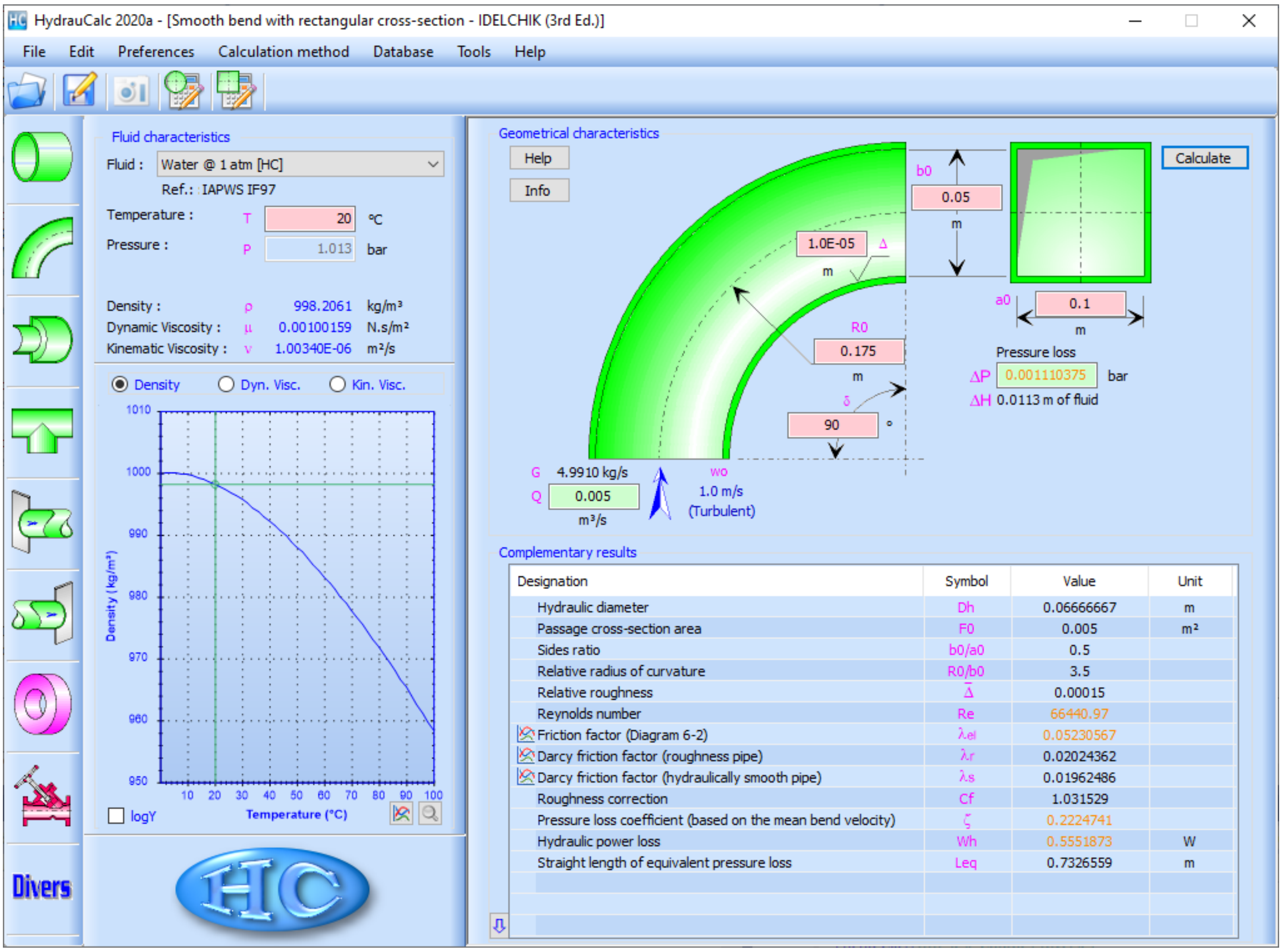
- flow regime: $Re \geq 3{\cdot}10^3$

■ case of relative radius of curvature greater than or equal to 3 ($R_0/b_0 \geq 3$)

- flow regime: $500 \leq Re \leq 38{\cdot}10^3$

  for Reynolds number 'Re' lower than 500 or greater than $38{\cdot}10^3$, the coefficient 'λ_{el}' is linearly extrapolated.

Example of application:



References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik
[2] Internal Flow System, Second Edition, D.S. Miller