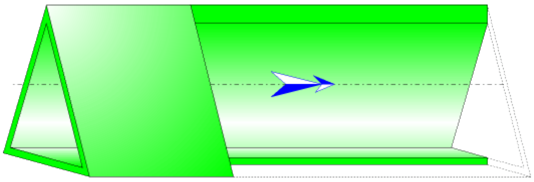
Straight Pipe
Triangular Cross-Section and Uniform Roughness Walls
(IDELCHIK)



Model description:

This model of component calculates the major head loss (pressure drop) of a horizontal straight pipe of triangular and constant cross-section.
In addition, the flow is assumed fully developed and stabilized.

The head loss is due to the friction of the fluid on the inner walls of the piping and is calculated with the Darcy formula. The roughness of the inner walls of the pipe is supposed uniform (pipe used by Nikuradse for its experimental data).

Darcy friction factor is determined:

- for laminar flow regime by the law of Hagen-Poiseuille (independent of the value of relative roughness),
- for turbulent flow regime by the Nikuradse equation (dependent of the value of relative roughness),
- for critical flow regime by interpolation between friction factors of laminar and turbulent flow.

Model formulation:

Half top angle (°):

$$\beta = \tan^{-1}\left(\frac{a_0}{2 \cdot h}\right)$$

Hydraulic diameter (m):

$$D_h = \frac{2 \cdot h}{1 + \sqrt{\frac{1}{\tan^2(\beta)} + 1}}$$

Cross-section area (m²):

$$F_0 = \frac{a_0}{2} \cdot h$$

Mean velocity (m/s):

$$w_0 = \frac{Q}{F_0}$$

Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

Fluid volume in the pipe (m³):

$$V = F_0 \cdot l$$

Fluid mass in the pipe (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$\mathrm{Re} = \frac{w_0 \cdot D_h}{\nu}$$

Relative roughness:

$$\overline{\Delta} = \frac{\Delta}{D_h}$$

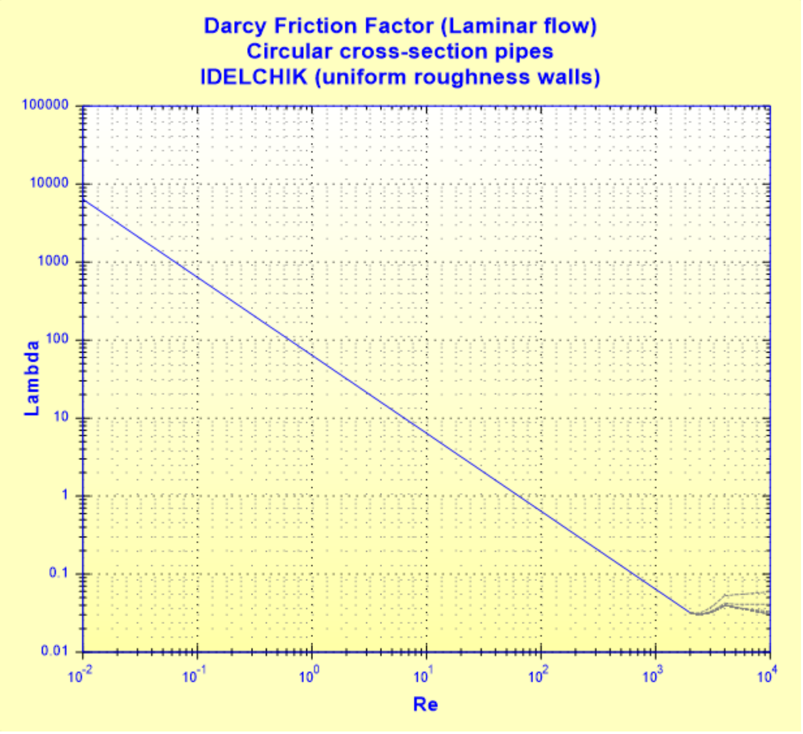
Darcy friction factor for circular cross-section:

- laminar flow regime ($\mathrm{Re} \leq 2000$):

  Hagen-Poiseuille law

$$\lambda_{circ} = \frac{64}{\mathrm{Re}}$$

([1] diagram 2.1)



■ turbulent flow regime – transition region and complete turbulence region (Re ≥ 4000):

Nikuradse equation

$$\lambda_{circ} = \frac{1}{\left[a_1 + b_1 \cdot \log(\mathrm{Re} \cdot \sqrt{\lambda}) + c_1 \cdot \log(\overline{\Delta})\right]^2}$$

([1] diagram 2.2)

where the values of a_1, b_1 and c_1 are given below:

$\overline{\Delta} \cdot \mathrm{Re} \cdot \sqrt{\lambda}$	a_1	b_1	c_1
3.6 - 10	-0.800	2.000	0.000
10 - 20	0.068	1.130	-0.870
20 - 40	1.538	0.000	-2.000
40 - 191.2	2.471	-0.588	-2.588
> 191.2	1.138	0.000	-2.000

Reynolds number at which pipe cease to be hydraulically smooth:

$$\mathrm{Re}'_{lim} = \frac{26.9}{\overline{\Delta}^{1.143}}$$

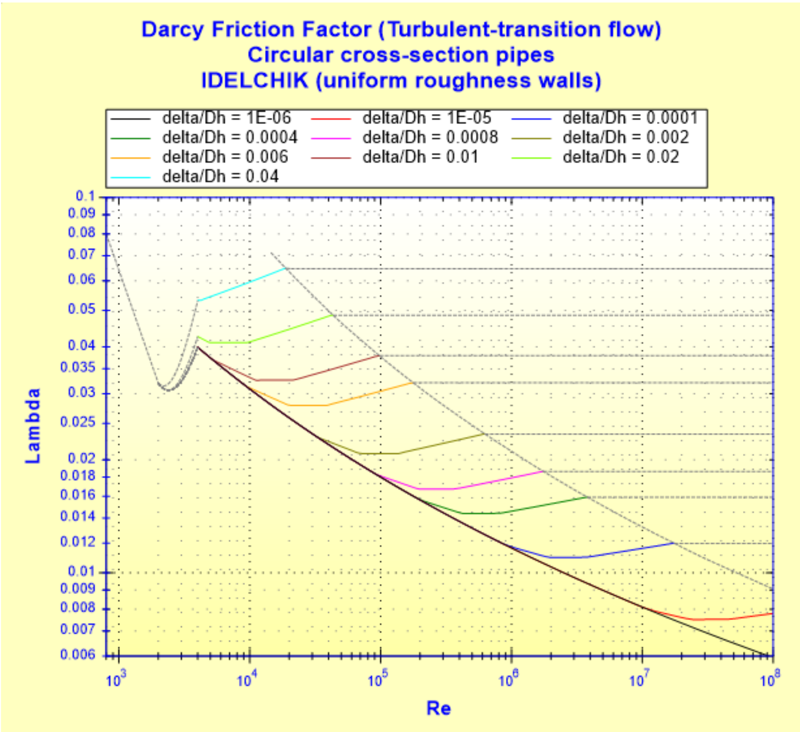
([1] §2.17)

Reynolds number corresponding to the beginning of complete turbulence:

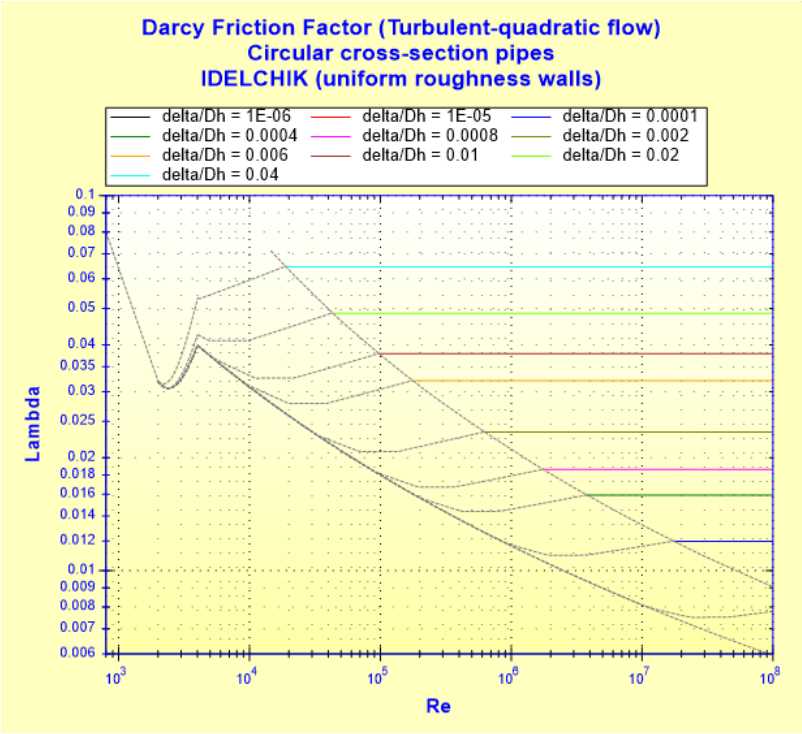
$$\mathrm{Re}''_{lim} = \frac{217.6 - 382.4 \cdot \log(\overline{\Delta})}{\overline{\Delta}}$$

([1] diagram 2.2)

Transition region



Complete turbulence region



- critical flow regime (2000 < Re < 4000):

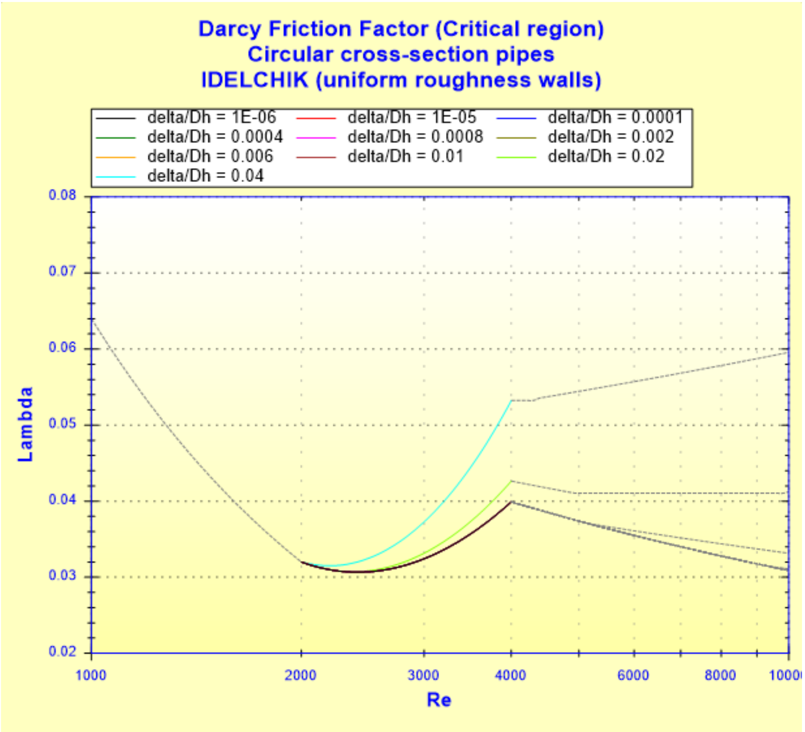
  linear interpolation

$$\lambda_{circ} = \lambda_L \cdot \left(1 - \frac{Re - 2000}{2000}\right) + \lambda_T \cdot \left(\frac{Re - 2000}{2000}\right)$$

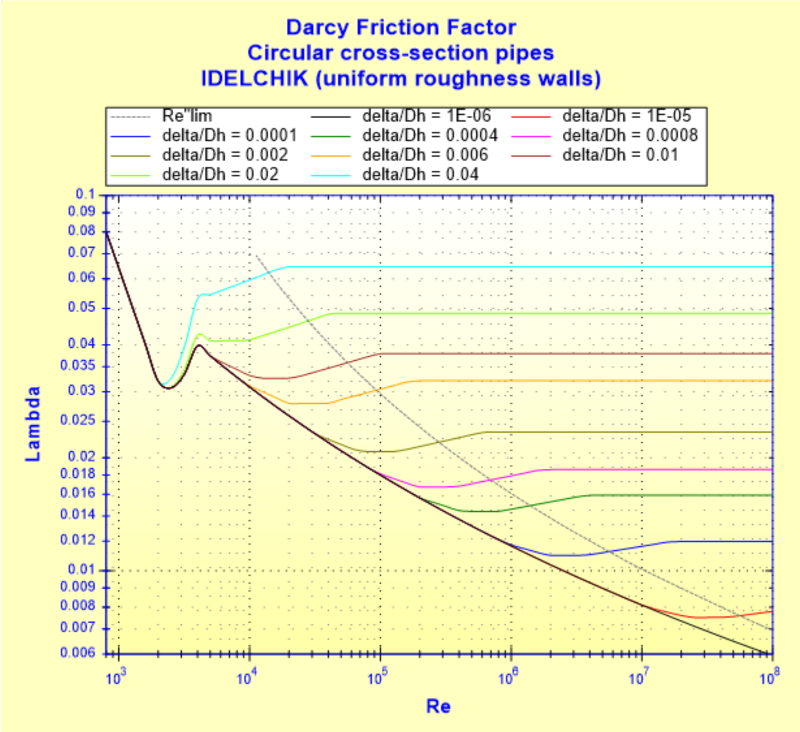
with:

λ_L = laminar friction coefficient obtained with Re = 2000

λ_T = turbulent friction coefficient obtained with Re = 4000



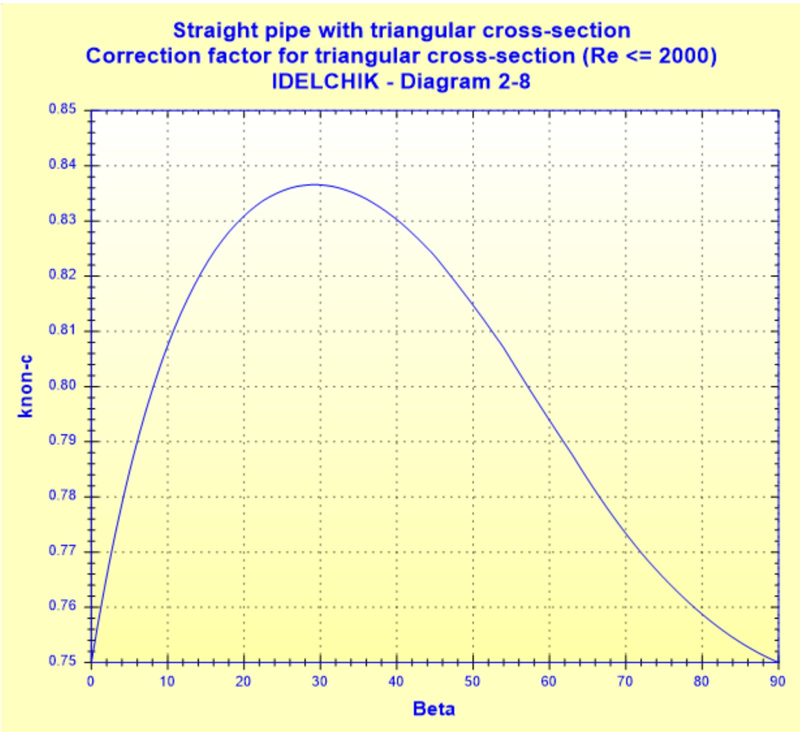
- all flow regimes:



Correction for Darcy friction factor for triangular cross-section:

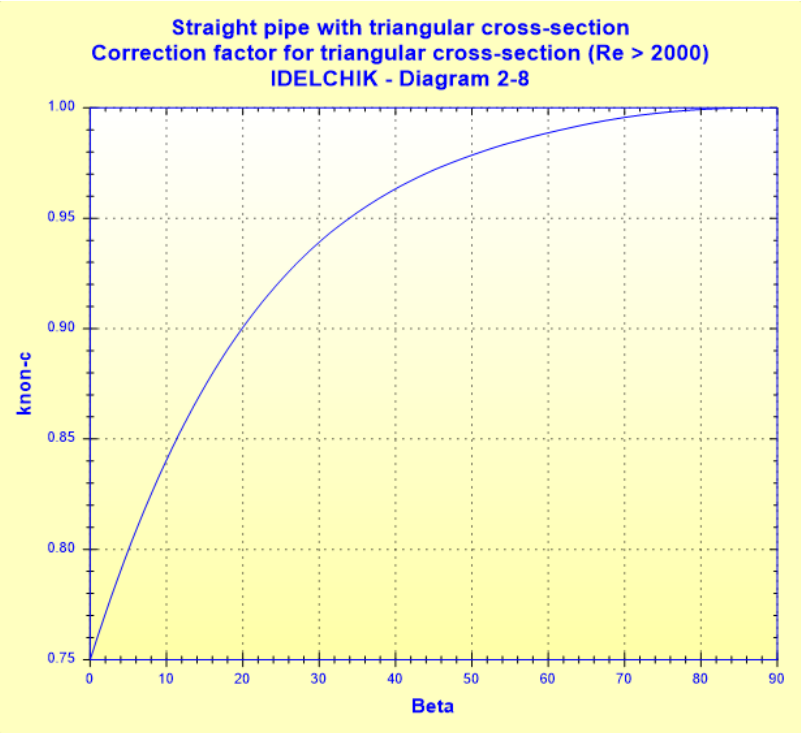
- laminar flow (Re $\leq$ 2000):

$$k_{non-c} = f(\beta)$$ ([1] diagram 2.8)



- turbulent flow (Re > 2000):

$$k_{non-c} = f(\beta)$$ ([1] diagram 2.8)



Darcy friction factor for triangular cross-section:

$$\lambda_{tria} = \lambda_{circ} \cdot k_{non-c}$$ ([1] diagram 2.8)

Pressure loss coefficient (based on the mean pipe velocity):

$$\zeta = \lambda_{tria} \cdot \frac{l}{D_h}$$ ([1] diagram 2.8)

Total pressure loss (Pa):

$$\Delta P = \zeta \cdot \frac{\rho \cdot w_0^2}{2}$$ ([1] diagram 2.8)

Total head loss of fluid (m):

$$\Delta H = \zeta \cdot \frac{w_0^2}{2 \cdot g}$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Symbols, Definitions, SI Units:

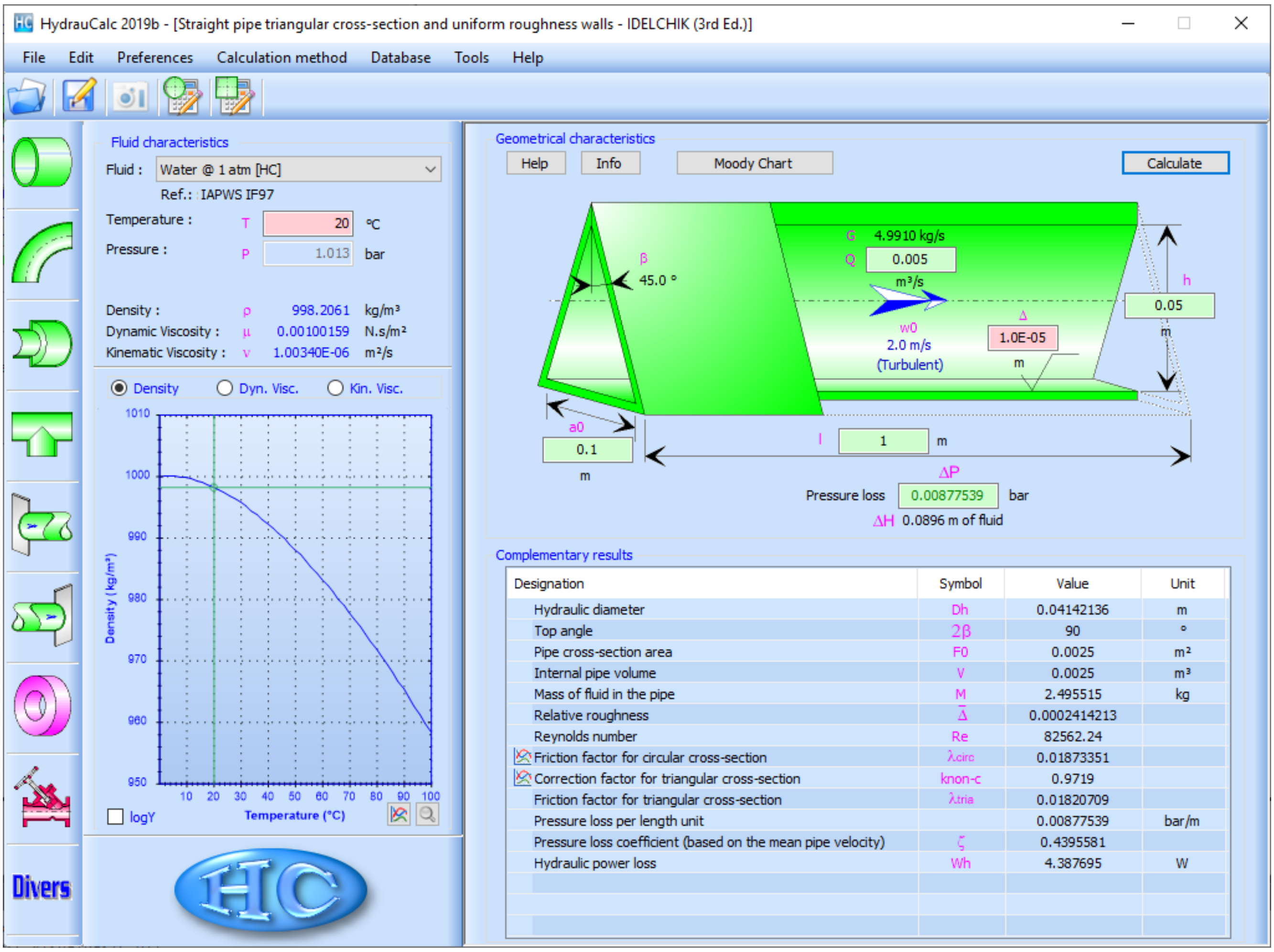
a_0	Cross-section base (m)
h	Cross-section height(m)
β	Half top angle (°)
D_h	Hydraulic diameter (m)
F_0	Cross-sectional area (m^2)
Q	Volume flow rate (m^3/s)
w_0	Mean velocity (m/s)

G	Mass flow rate (kg/s)
l	Pipe length (m)
V	Fluid volume in the pipe (m^3)
M	Fluid mass in the pipe (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\bar{\Delta}$	Relative roughness of walls ()
λ_{circ}	Darcy friction factor for circular cross-section ()
Re'_{lim}	Limiting Reynolds number for hydraulically smooth law ()
Re''_{lim}	Limiting Reynolds number for quadratic law ()
k_{non-c}	Correction for Darcy friction factor for triangular cross-section ()
λ_{tria}	Darcy friction factor for triangular cross-section ()
ζ	Pressure loss coefficient (based on the mean pipe velocity) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- any flow regime: laminar, critical and turbulent ($Re \leq 10^8$)
- relative roughness $\bar{\Delta} \leq 0.05$
- stabilized flow

Example of input data and results:



References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik (2008)

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