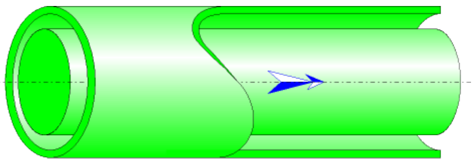
Straight Pipe
Annular Cross-Section and Nonuniform Roughness Walls
(IDELCHIK)



Model description:

This model of component calculates the major head loss (pressure drop) of a horizontal straight pipe of annular and constant cross-section.
In addition, the flow is assumed fully developed and stabilized.

The head loss is due to the friction of the fluid on the inner walls of the piping and is calculated with the Darcy formula. The roughness of the inner walls of the pipe is supposed nonuniform (commercial pipe).

Darcy friction factor is determined:

- for laminar flow regime by the law of Hagen-Poiseuille (independent of the value of relative roughness),
- for turbulent flow regime by the implicit Colebrook-White equation (dependent of the value of relative roughness),
- for critical flow regime by interpolation between friction factors of laminar and turbulent flow.

Model formulation:

Hydraulic diameter (m):

$$D_h = D_0 - d$$

Cross-section area (m²):

$$F_0 = \pi \cdot \frac{D_0^2 - d^2}{4}$$

Mean velocity (m/s):

$$w_0 = \frac{Q}{F_0}$$

Mass flow rate (kg/s):

$$G = Q \cdot \rho$$

Fluid volume in the pipe (m^3):

$$V = F_0 \cdot l$$

Fluid mass in the pipe (kg):

$$M = V \cdot \rho$$

Reynolds number:

$$\mathrm{Re} = \frac{w_0 \cdot D_h}{\nu}$$

Relative roughness:

$$\bar{\Delta} = \frac{\Delta}{D_h}$$

Relative eccentricity:

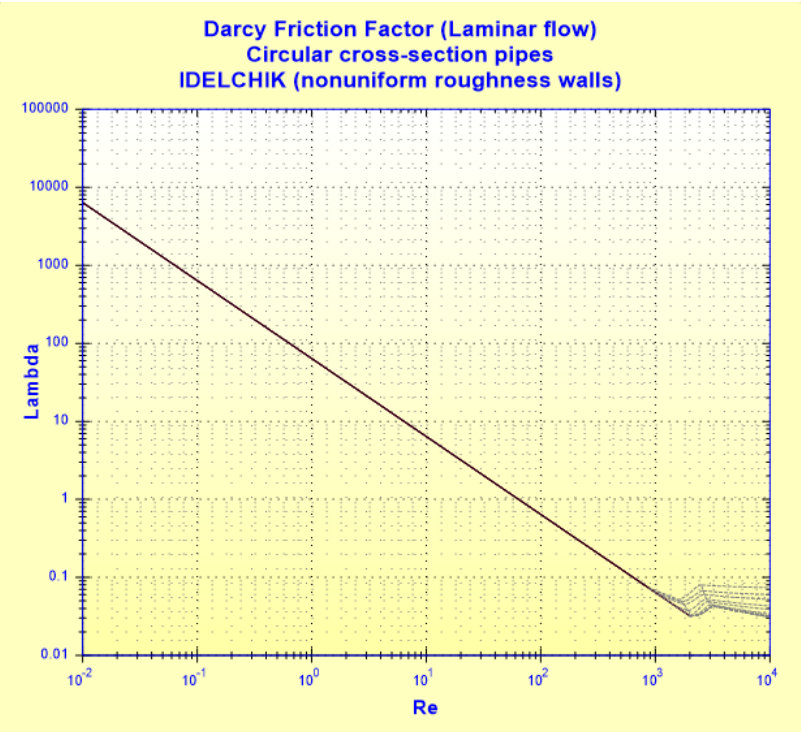
$$\bar{e} = \frac{2 \cdot e}{D_0 - d}$$

Darcy friction factor for circular cross-section:

- laminar flow regime ($\mathrm{Re} \le \mathrm{Re}_0$):

  Hagen-Poiseuille law

$$\lambda_{circ} = \frac{64}{\mathrm{Re}}$$ ([1] diagram 2.1)



- turbulent flow regime – transition region and complete turbulence region ($\mathrm{Re} \ge \mathrm{Re}_2$):

  Colebrook-White equation

$$\lambda_{circ} = \frac{1}{\left[2 \cdot \log\left(\frac{2.51}{Re \cdot \sqrt{\lambda}} + \frac{\overline{\Delta}}{3.7}\right)\right]^2}$$

([1] diagram 2.4)

Reynolds number at which pipe cease to be hydraulically smooth:

$$Re'_{lim} = \frac{15}{\overline{\Delta}}$$

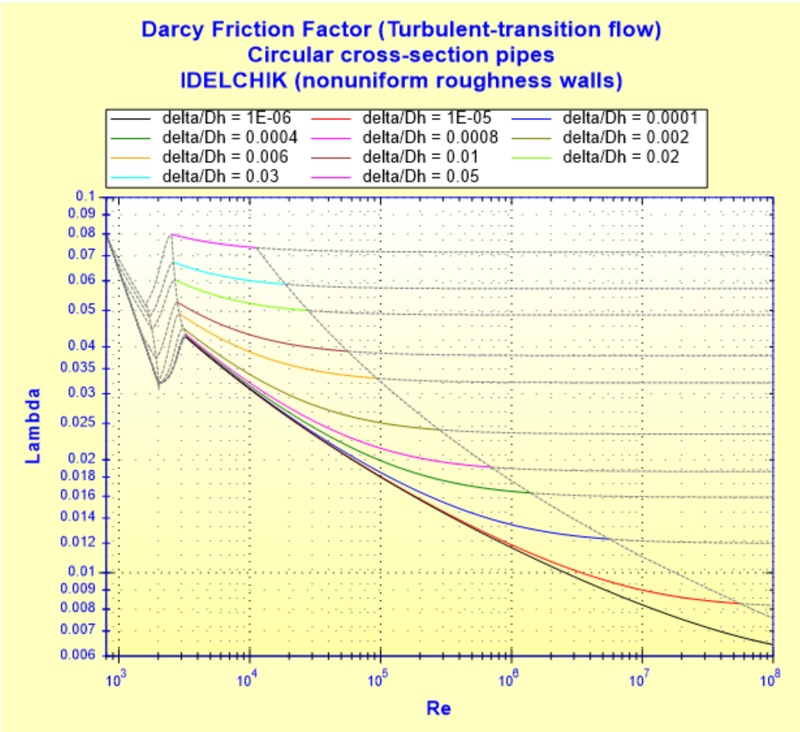
([1] §2.23)

Reynolds number corresponding to the beginning of complete turbulence:

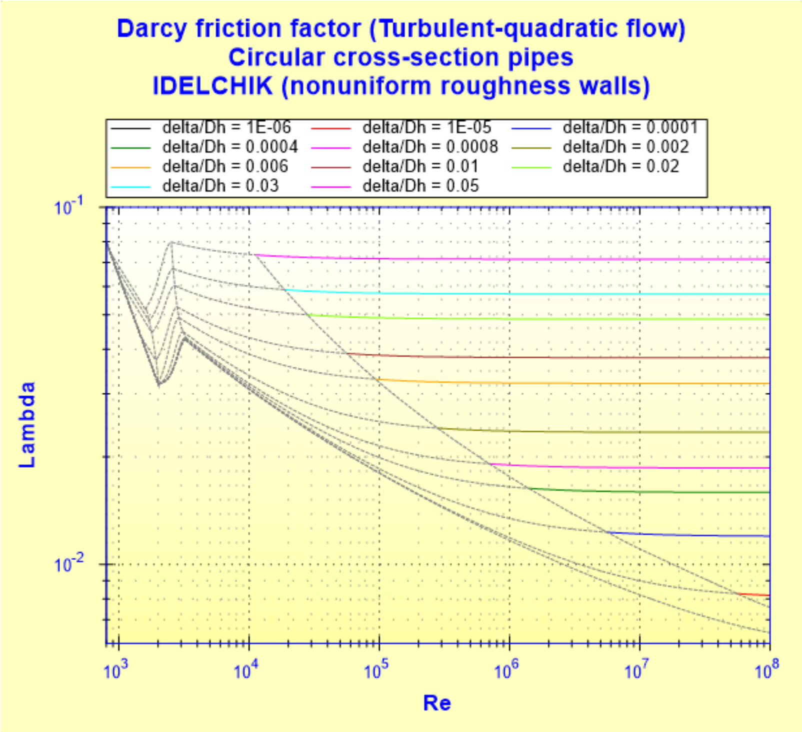
$$Re''_{lim} = \frac{560}{\overline{\Delta}}$$

([1] diagram 2.4)

Transition region



Complete turbulence region



■ critical flow regime ($Re_0 < Re < Re_2$):

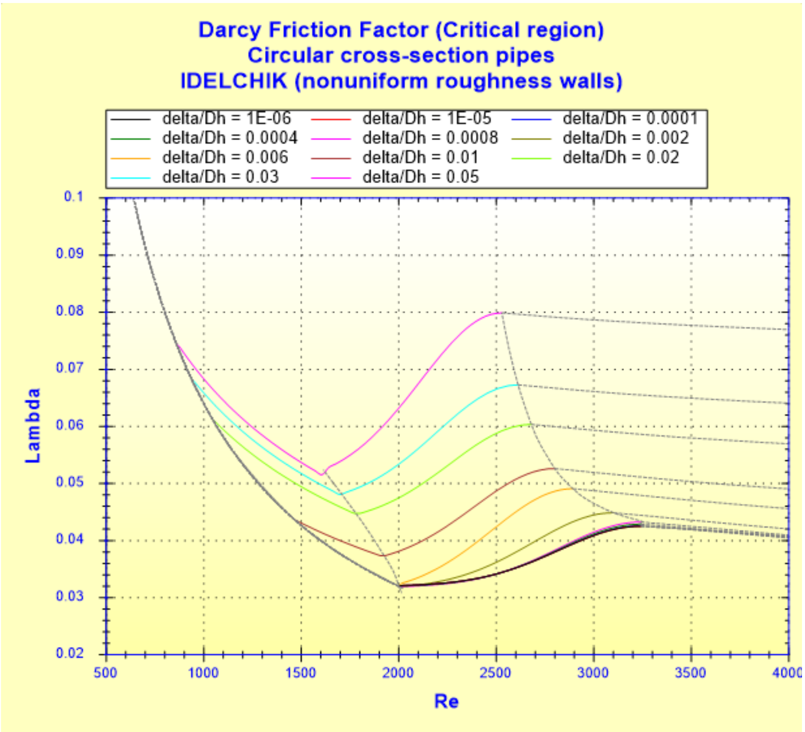
$$\lambda_{circ} = f(\mathrm{Re}, \overline{\Delta})$$ ([1] diagram 2.3)

Reynolds number of start of critical zone:

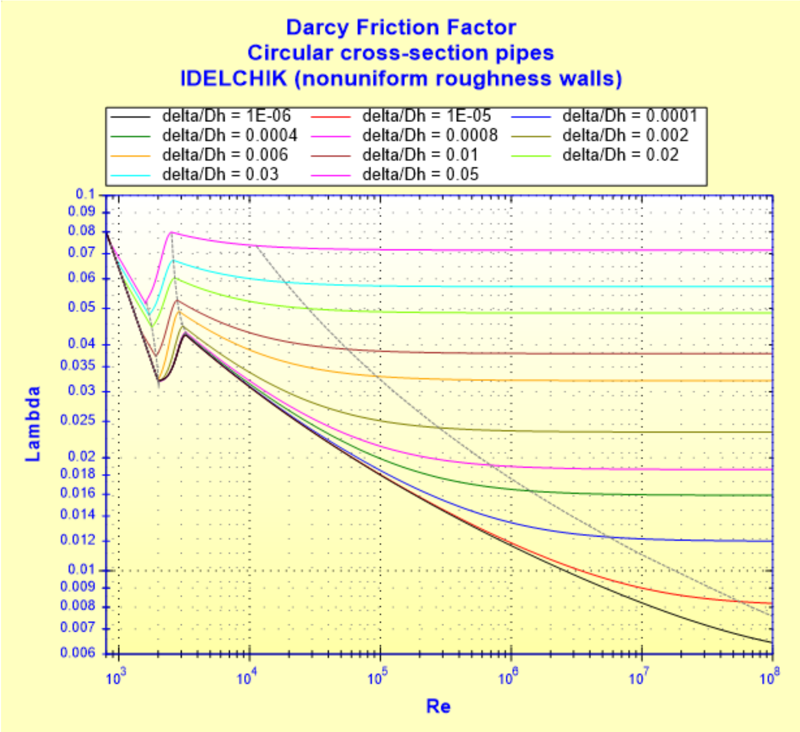
$$\mathrm{Re}_0 = 754\exp\left(\frac{0.0065}{\overline{\Delta}}\right)$$ ([1] §2.21)

Reynolds number at end of critical zone:

$$\mathrm{Re}_2 = 2090\left(\frac{1}{\overline{\Delta}}\right)^{0.0635}$$ ([1] §2.22)



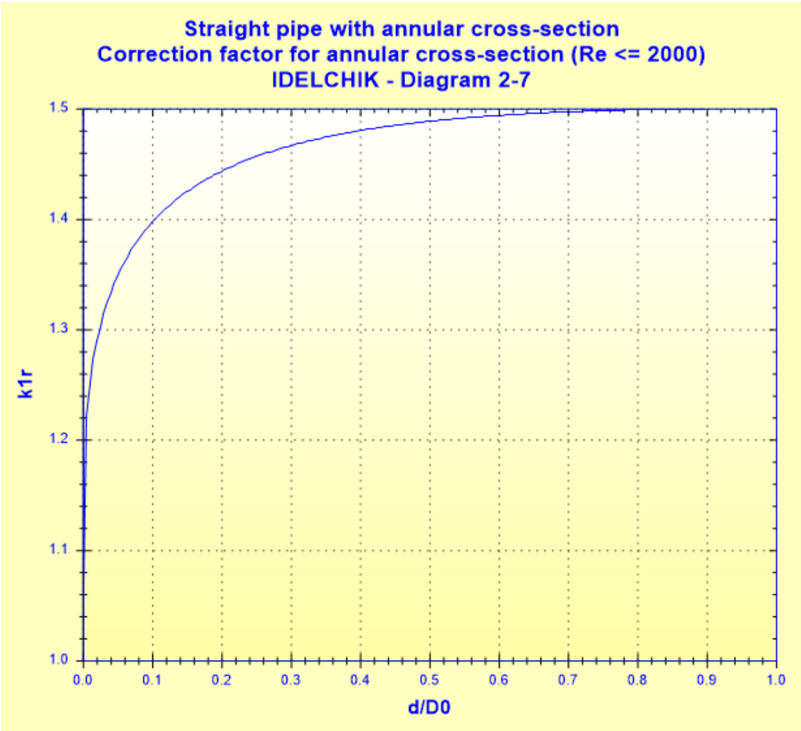
■ all flow regimes:



Correction for Darcy friction factor for annular cross-section:

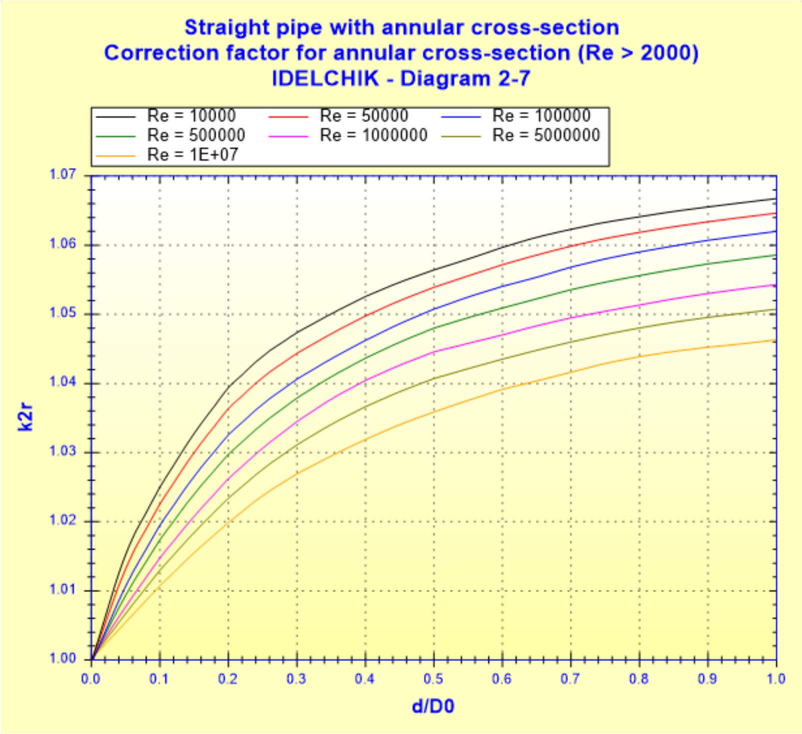
■ laminar flow (Re ≤ 2000):

$$k_{1r} = f\left(d/D_0\right)$$ ([1] diagram 2.7)



■ turbulent flow (Re > 2000):

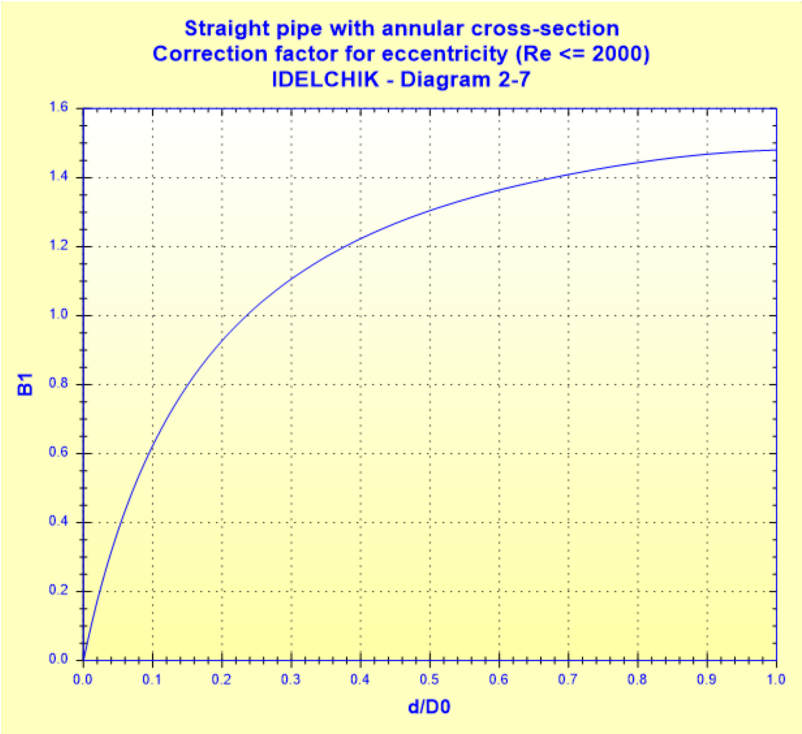
$$k_{2r} = f\left(d/D_0, \text{Re}\right)$$ ([1] diagram 2.7)



Correction for Darcy friction factor for axis eccentricity:

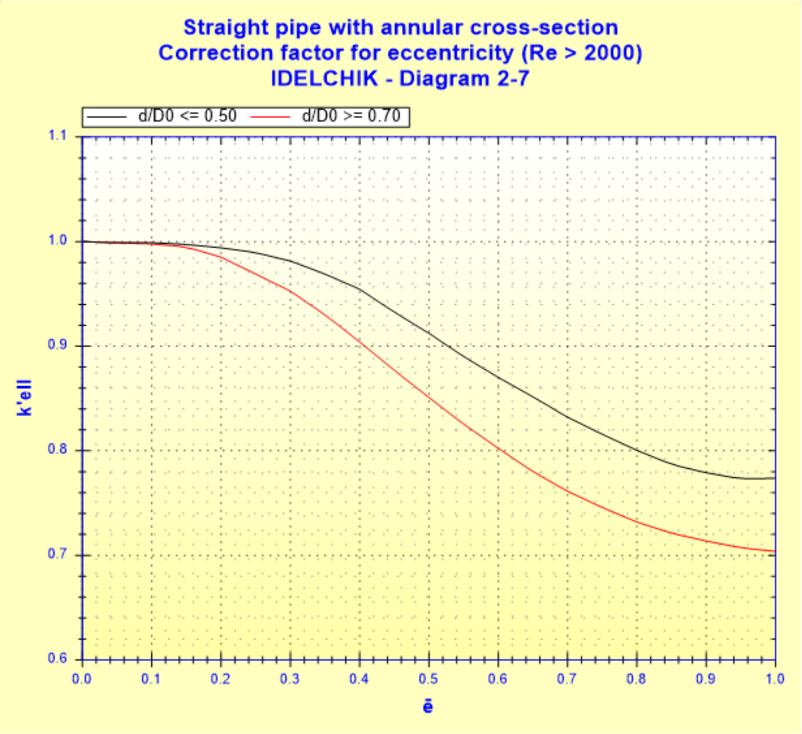
- laminar flow ($Re \le 2000$):

$$B_1 = f\left(d/D_0\right)$$ ([1] diagram 2.7)



- turbulent flow ($Re > 2000$):

$$k'_{ell} = f\left(\bar{e}, d/D_0\right)$$ ([1] diagram 2.7)



Darcy friction factor for annular cross-section:

- laminar flow (Re $\leq$ 2000):

$$\lambda_{annu} = \lambda_{circ} \cdot k_{1r} \cdot B_1$$

- turbulent flow (Re $>$ 2000):

$$\lambda_{annu} = \lambda_{circ} \cdot k_{2r} \cdot k'_{ell}$$

Total correction for Darcy friction factor for noncircular cross-section:

- laminar flow (Re $\leq$ 2000):

$$k_{non-c} = k_{1r} \cdot B_1$$

- turbulent flow (Re $>$ 2000):

$$k_{non-c} = k_{2r} \cdot k'_{ell}$$

Pressure loss coefficient (based on the mean pipe velocity):

$$\zeta = \lambda_{circ} \cdot k_{non-c} \cdot \frac{l}{D_h}$$

([1] diagram 2.7)

Total pressure loss (Pa):

$$\Delta P = \zeta \cdot \frac{\rho \cdot w_0^2}{2}$$

([1] diagram 2.7)

Total head loss of fluid (m):

$$\Delta H = \zeta \cdot \frac{w_0^2}{2 \cdot g}$$

Hydraulic power loss (W):

$$Wh = \Delta P \cdot Q$$

Symbols, Definitions, SI Units:

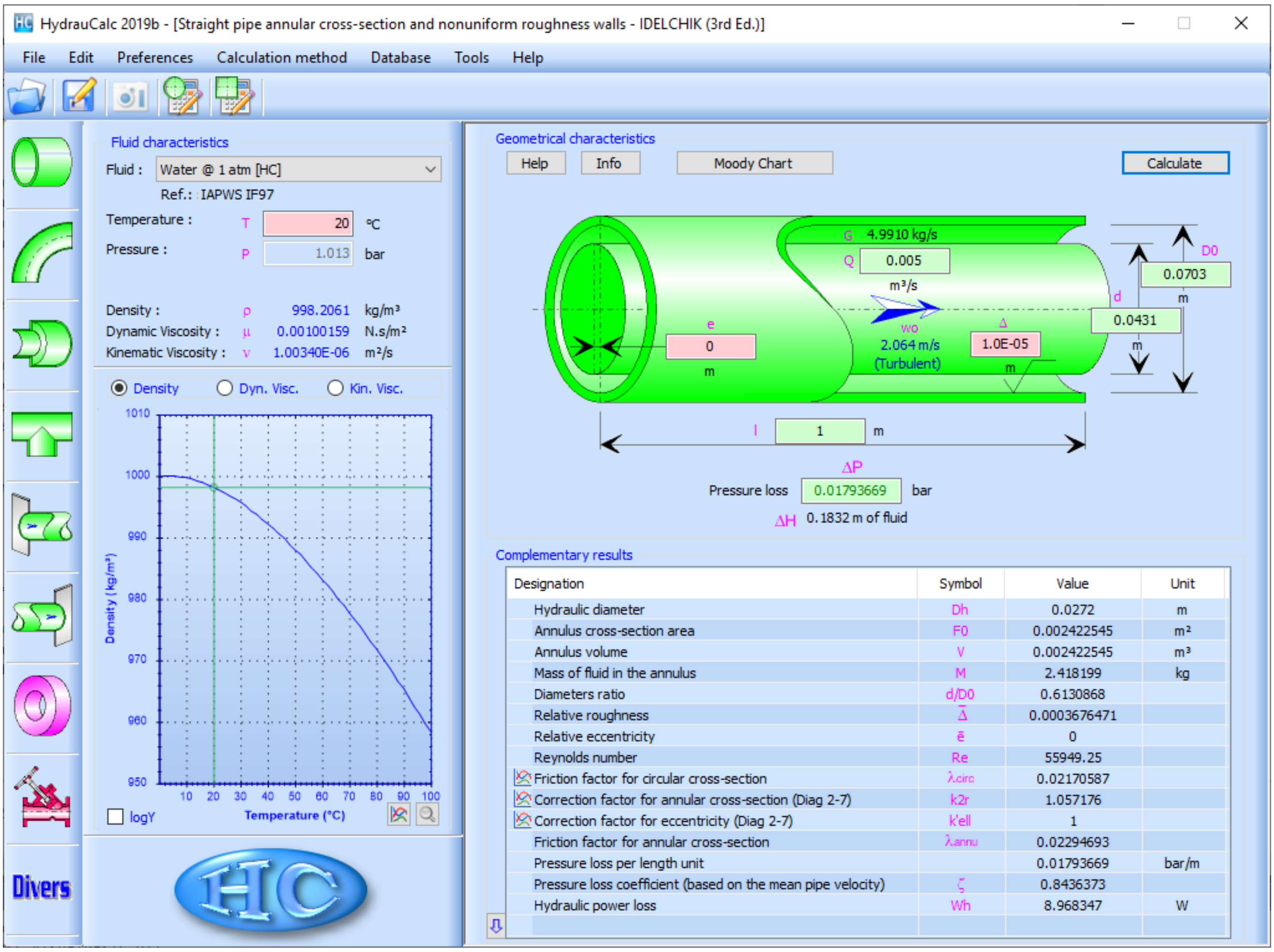
D_0	Annulus outer diameter (m)
d	Annulus inner diameter (m)
D_h	Hydraulic diameter (m)
F_0	Cross-sectional area (m²)
Q	Volume flow rate (m³/s)
w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
l	Pipe length (m)
V	Fluid volume in the pipe (m³)
M	Fluid mass in the pipe (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\bar{\Delta}$	Relative roughness of walls ()
e	Pipes eccentricity (m)
$\bar{e}$	Relative eccentricity ()
λ_{circ}	Darcy friction factor for circular cross-section ()
Re_0	Reynolds number of start of critical zone ()
Re_2	Reynolds number at end of critical zone ()
Re'_{lim}	Limiting Reynolds number for hydraulically smooth law ()
Re''_{lim}	Limiting Reynolds number for quadratic law ()
K_{1r}	Correction for noncircular cross-section (laminar regime) ()
K_{2r}	Correction for noncircular cross-section (turbulent regime) ()
B_1	Correction for eccentricity (laminar regime) ()
K'_{ell}	Correction for eccentricity (turbulent regime) ()
λ_{annu}	Darcy friction factor for annular cross-section ()
$k_{non\text{-}c}$	Correction for Darcy friction factor for noncircular cross-section ()
ζ	Pressure loss coefficient (based on the mean pipe velocity) ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m³)
ν	Fluid kinematic viscosity (m²/s)
g	Gravitational acceleration (m/s²)

Validity range:

- any flow regime: laminar, critical and turbulent ($Re \leq 10^8$)
  note: for Reynolds number lower than 10^4 or greater than 10^7, the correction factor 'k2r' is extrapolated
- relative roughness $\bar{\Delta} \leq 0.05$

- stabilized flow

Example of input data and results:



References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik (2008)
