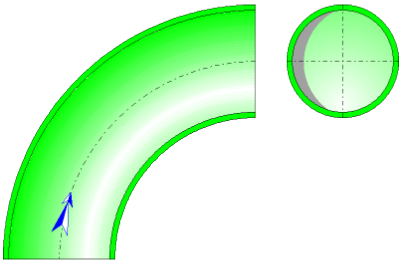
Smooth Bend
Circular Cross-Section
(IDELCHIK)



Model description:

This model of component calculates the head loss (pressure drop) of a bend smoothly curved whose cross-section is circular and constant. In addition, the flow is assumed fully developed and stabilized à the entrance bend.

Model formulation:

Hydraulic diameter (m):

$$\boxed{\mathrm{D_h} = D_0}$$

Cross-section area (m^2):

$$\boxed{\mathrm{F_0} = \pi \cdot \frac{{D_0}^2}{4}}$$

Length measured along the axis (m):

$$\boxed{\mathrm{l} = 2 \cdot \pi \cdot R_0 \cdot \frac{\delta}{360}}$$

Mean velocity (m/s):

$$\boxed{w_0 = \frac{Q}{F_0}}$$

Mass flow rate (kg/s):

$$\boxed{G = Q \cdot \rho}$$

Fluid volume (m^3):

$$\boxed{V = F_0 \cdot l}$$

Fluid mass (kg):

$$\boxed{M = V \cdot \rho}$$

Reynolds number:

$$\boxed{\mathrm{Re} = \frac{w_0 \cdot D_h}{\nu}}$$

Relative roughness:

$$\boxed{\bar{\Delta} = \frac{\Delta}{D_0}}$$

■ Case of relative radius of curvature lower than 3 ($R_0/D_0 < 3$) ([1] diagram 6.1)

Coefficient of effect of the roughness:

$$\boxed{k_\Delta = f\left(\frac{R_0}{D_0}, \mathrm{Re}, \bar{\Delta}\right)}$$

([1] diagram 6.1)

- $0.50 \le R_0/D_0 \le 0.55$

$\bar{\Delta}$	Re	
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$
0	1.0	1.0
0 - 0.001	1.0	$1 + 0.5 \cdot 10^3 \cdot \bar{\Delta}$
> 0.001	1.0	1.5

- $R_0/D_0 > 0.55$

$\bar{\Delta}$	Re		
	$3 \cdot 10^3$ - $4 \cdot 10^4$	> $4 \cdot 10^4$ - $2 \cdot 10^5$	> $2 \cdot 10^5$
0	1.0	1.0	1.0
0 - 0.001	1.0	$\lambda_\Delta / \lambda_{sm}$	$1 + 10^3 \cdot \bar{\Delta}$
> 0.001	1.0	2.0	2.0

with:

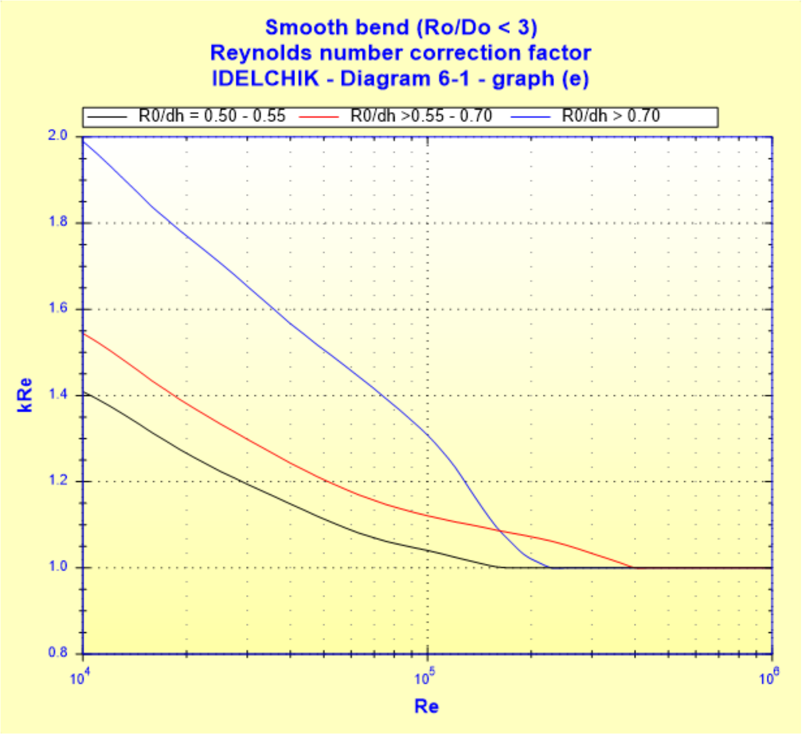
λ_{sm} : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

λ_Δ : Darcy friction factor for rough pipe ($\bar{\Delta} = \Delta/D_h$) at Re

Coefficient of effect of the Reynolds number ($\mathrm{Re} \ge 10^4$):

$$\boxed{k_{\mathrm{Re}} = f\left(\mathrm{Re}, \frac{R_0}{D_h}\right)}$$

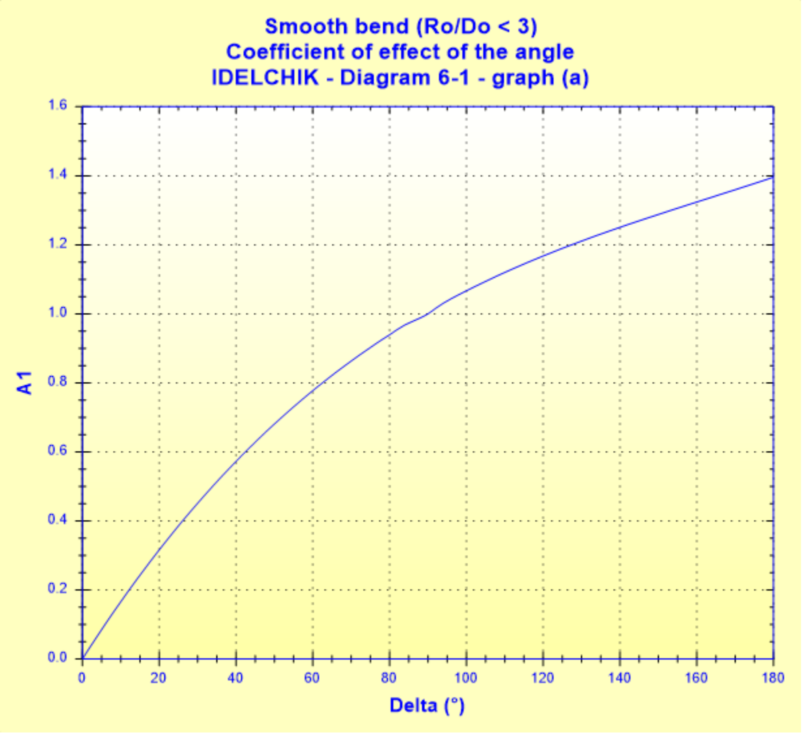
([1] diagram 6.1)



Coefficient of effect of the angle:

$$\boxed{A1 = f(\delta)}$$

([1] diagram 6.1)

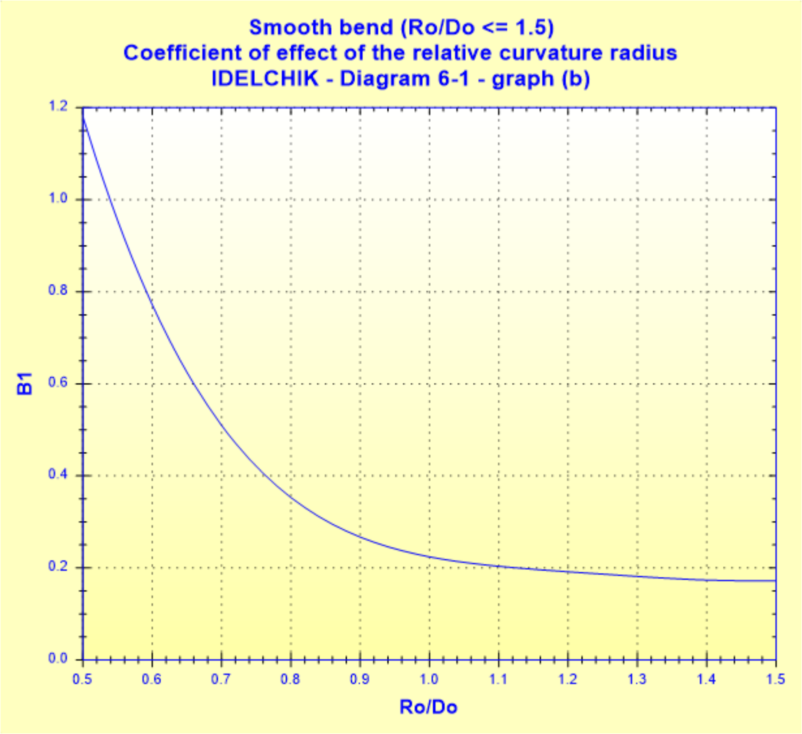


Coefficient of effect of the relative curvature radius:

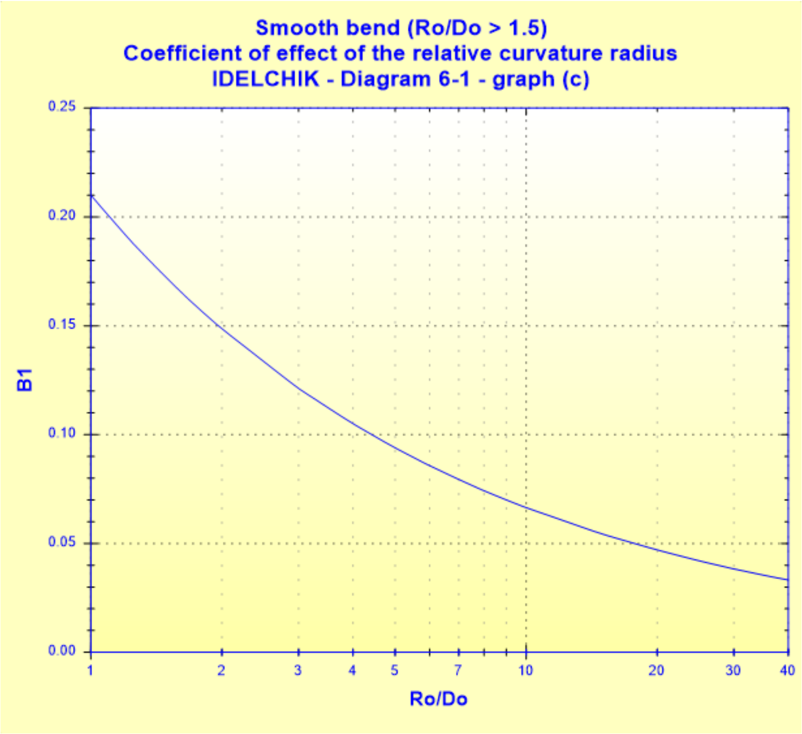
$$\boxed{B1 = f\left(\frac{R_0}{D_h}\right)}$$

([1] diagram 6.1)

- $0.5 \leq R_0/D_0 \leq 1.5$



- $R_0/D_0 > 1.5$



Coefficient of effect of the relative elongation of the cross section:

$\boxed{C1 = 1}$ ([1] diagram 6.1)

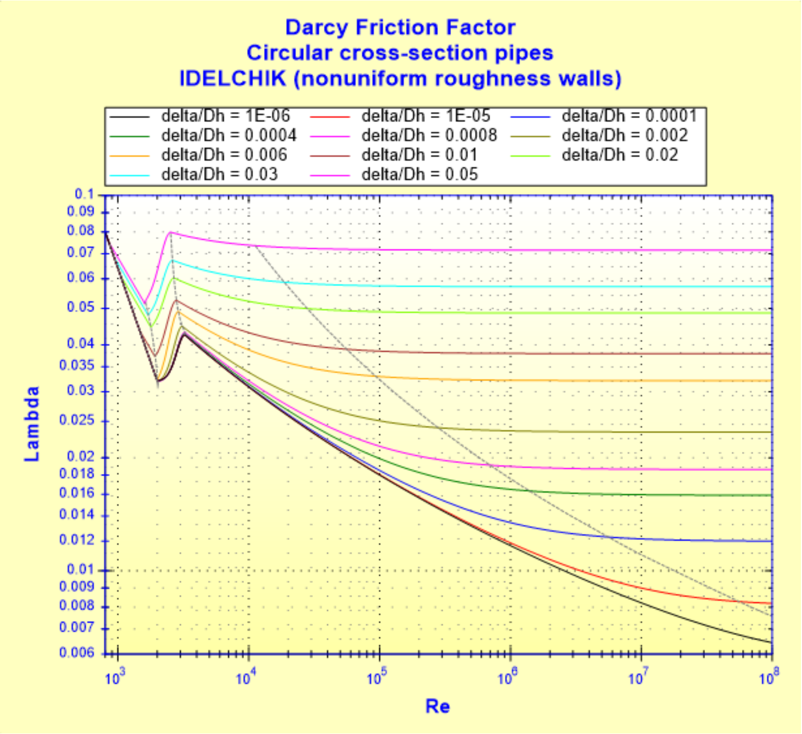
Coefficient of local resistance:

$\boxed{\zeta_{loc} = A1 \cdot B1 \cdot C1}$ ([1] diagram 6.1)

Darcy friction factor:

$$\boxed{\lambda = f\left(\mathrm{Re}, \frac{\Delta}{D_h}\right)}$$

See Straight Pipe - Circular Cross-Section and Nonuniform Roughness Walls (IDELCHIK)



Pressure loss friction factor:

$$\boxed{\zeta_{fr} = 0.0175 \cdot \delta \cdot \lambda \cdot \frac{R_0}{D_h}}$$

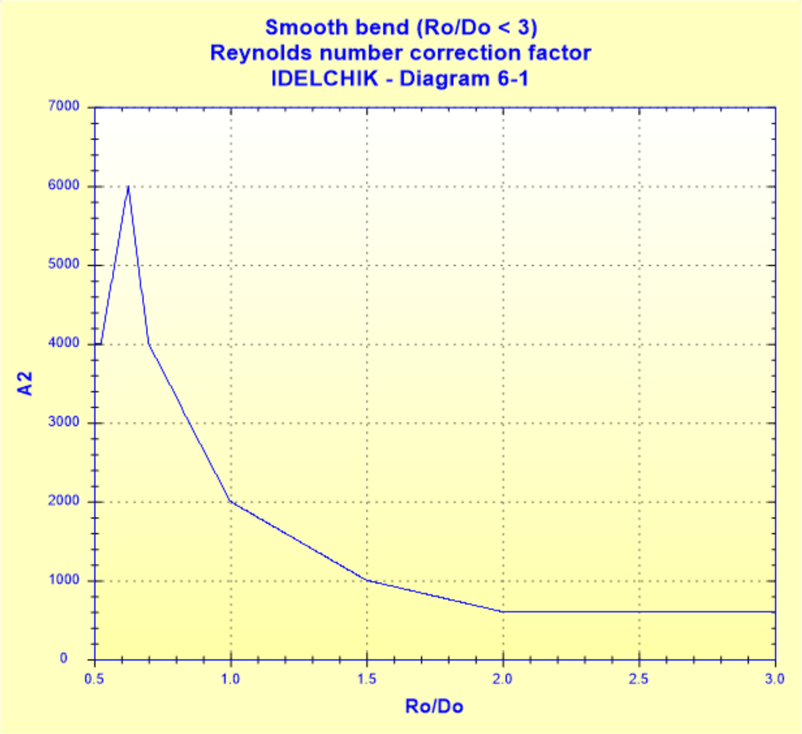
([1] diagram 6.1)

Reynolds number correction factor that depends on the relative curvature radius:

$$\boxed{A2 = f\left(\frac{R_0}{D_0}\right)}$$

([1] diagram 6.1)

R_0/D_0	0.50 - 0.55	>0.55 - 0.70	>0.70 - 1.0	>1.0 - 2.0	>2.0
$A2 \times 10^{-3}$	4.0	6.0	4.0 - 2.0	1.0	0.6



Total pressure loss coefficient:

- $Re \geq 10^4$

$$\boxed{\zeta = k_\Delta \cdot k_{Re} \cdot \zeta_{loc} + \zeta_{fr}}$$ ([1] diagram 6.1)

- $3 \cdot 10^3 < Re < 10^4$

$$\boxed{\zeta = \frac{A2}{Re} + \zeta_{loc} + \zeta_{fr}}$$ ([1] diagram 6.1)

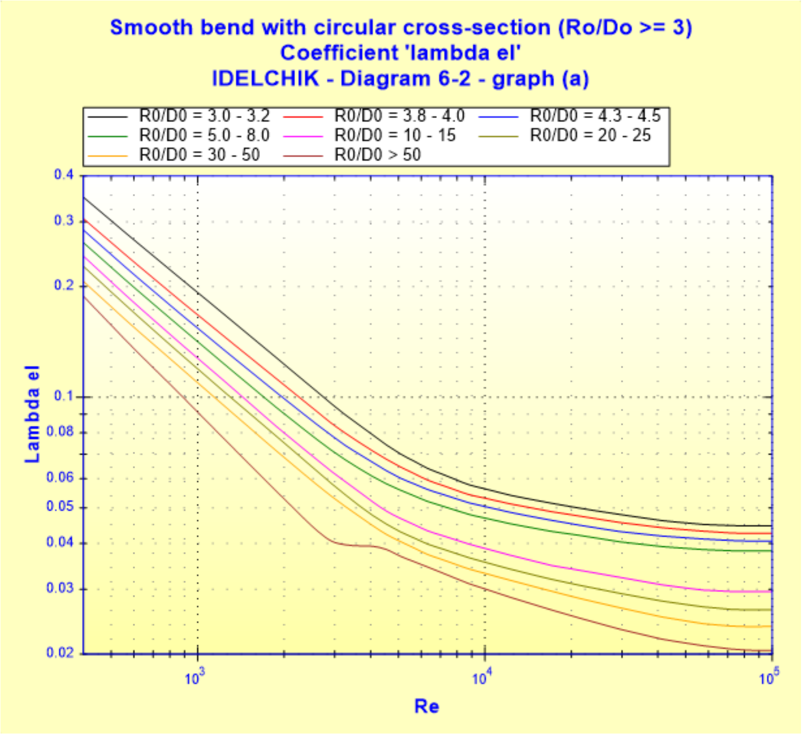
Straight length of equivalent pressure loss (m):

$$\boxed{L_{eq} = \zeta \cdot \frac{D_0}{\lambda}}$$

■ Case of relative radius of curvature greater than or equal to 3 ($R_0/D_0 \geq 3$) ([1] diagram 6.2)

Friction factor smooth wall:

$$\boxed{\lambda_{el} = f\left(Re, \frac{R_0}{D_0}\right)}$$ ([1] diagram 6.2)



Roughness correction factor:

$$\boxed{C_f = \frac{\lambda_r}{\lambda_s}}$$

([2] equation 9.3)

with:

λ_r : Darcy friction factor for rough pipe ($\bar{\Delta} = \Delta / D_h$) at Re

λ_s : Darcy friction factor for hydraulically smooth pipe ($\bar{\Delta} = 0$) at Re

Pressure loss coefficient:

$$\boxed{\zeta = 0.0175 \cdot \delta \cdot \lambda_{el} \cdot C_f \cdot \frac{R_0}{D_h}}$$

([1] diagram 6.2 & [2] equation 9.3)

Straight length of equivalent pressure loss (m):

$$\boxed{L_{eq} = \zeta \cdot \frac{D_0}{\lambda_{el}}}$$

Total pressure loss (Pa):

$$\boxed{\Delta P = \zeta \cdot \frac{\rho \cdot w_0^{\ 2}}{2}}$$

([1] diagram 6.1 - 6.2)

Total head loss of fluid (m):

$$\boxed{\Delta H = \zeta \cdot \frac{w_0^{\ 2}}{2 \cdot g}}$$

Hydraulic power loss (W):

$$\boxed{Wh = \Delta P \cdot Q}$$

Symbols, Definitions, SI Units:

D_h	Bend hydraulic diameter (m)
D_0	Bend internal diameter (m)
F_0	Cross-sectional area (m^2)
l	Length measured along the axis (m)
R_0	Radius of curvature (m)
δ	Curvature angle (°)
Q	Volume flow rate (m^3/s)
w_0	Mean velocity (m/s)
G	Mass flow rate (kg/s)
V	Fluid volume (m^3)
M	Fluid mass (kg)
Re	Reynolds number ()
Δ	Absolute roughness of walls (m)
$\overline{\Delta}$	Relative roughness of walls ()
k_{Δ}	Coefficient that allows for the effect of the roughness
k_{Re}	Coefficient that allows for the effect of the Reynolds number
A1	Coefficient that allows for the effect of the angle
B1	Coefficient that allows for the effect of the relative curvature radius
C1	Coefficient that allows for the effect of the relative elongation of the cross section
ζ_{loc}	Coefficient of local resistance ()
λ	Darcy friction coefficient ()
ζ_{fr}	Pressure loss friction factor ()
ζ	Total pressure loss coefficient (based on the mean velocity in the bend) ()
L_{eq}	Straight length of equivalent pressure loss (m)
λ_{el}	Friction coefficient ()
C_f	Roughness correction factor ()
ΔP	Total pressure loss (Pa)
ΔH	Total head loss of fluid (m)
Wh	Hydraulic power loss (W)
ρ	Fluid density (kg/m^3)
ν	Fluid kinematic viscosity (m^2/s)
g	Gravitational acceleration (m/s^2)

Validity range:

- stabilized flow upstream bend
- length of the straight section downstream: $\geq 10\ D_0$
- curvature angle: 0 to 180°

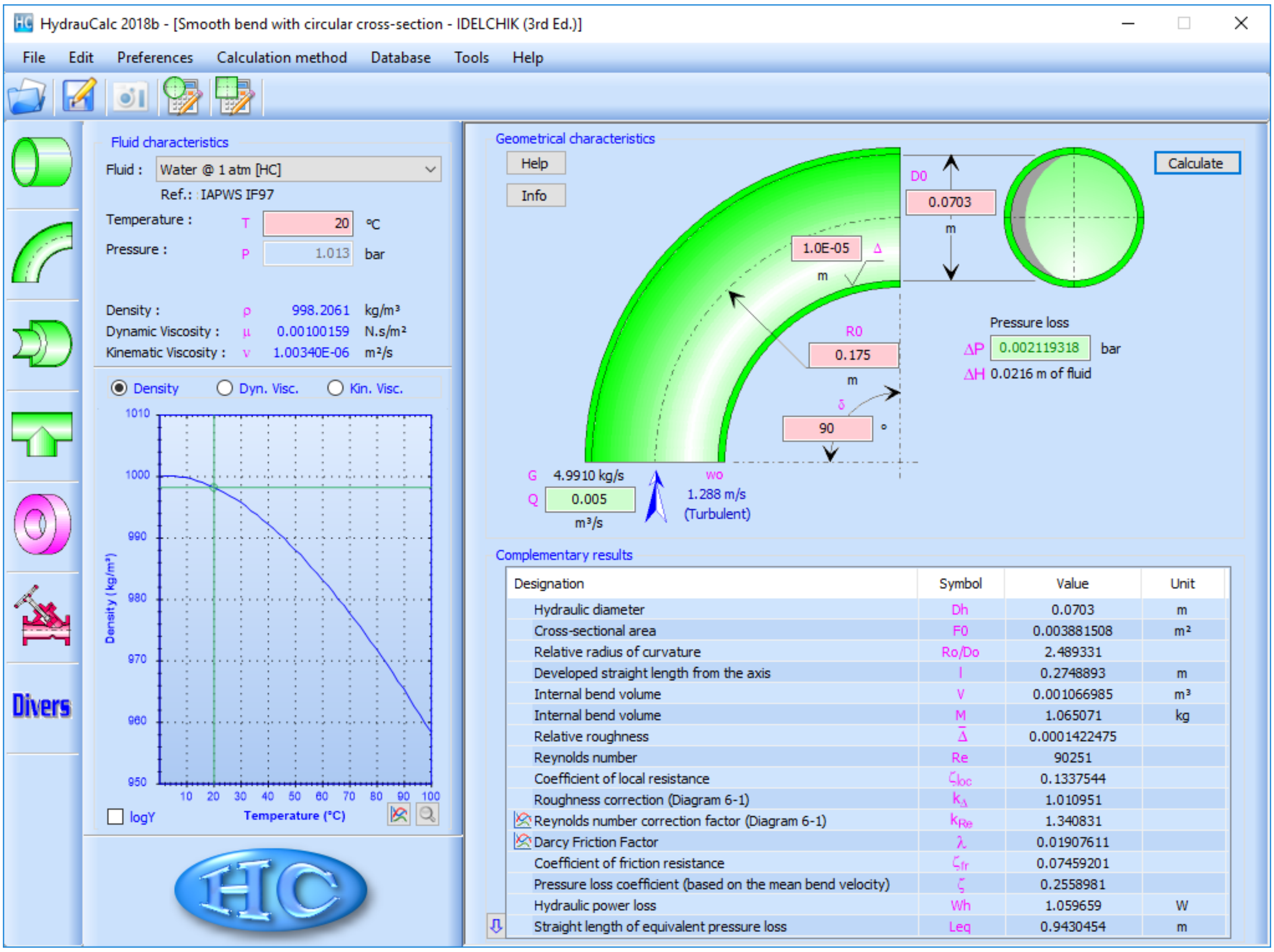
- ■ case of relative radius of curvature lower than 3 ($R_0/D_0 < 3$)
  - flow regime: $Re \geq 3 \cdot 10^3$

- ■ case of relative radius of curvature greater than or equal to 3 ($R_0/D_0 \geq 3$)

- flow regime: $400 \le Re \le 10^5$

  for Reynolds number 'Re' lower than 400 or greater than 10^5, the coefficient 'λ_{el}' is linearly extrapolated.

Example of application:



Fluid characteristics

Fluid : Water @ 1 atm [HC]

Ref.: IAPWS IF97

Temperature : T 20 °C

Pressure : P 1.013 bar

Density : ρ 998.2061 kg/m³

Dynamic Viscosity : µ 0.00100159 N.s/m²

Kinematic Viscosity : ν 1.00340E-06 m²/s

● Density ○ Dyn. Visc. ○ Kin. Visc.

Geometrical characteristics

D0 0.0703 m

Δ 1.0E-05 m

R0 0.175 m

δ 90 °

Pressure loss

ΔP 0.002119318 bar

ΔH 0.0216 m of fluid

G 4.9910 kg/s

Q 0.005 m³/s

wo 1.288 m/s (Turbulent)

Complementary results

Designation	Symbol	Value	Unit
Hydraulic diameter	Dh	0.0703	m
Cross-sectional area	F0	0.003881508	m²
Relative radius of curvature	Ro/Do	2.489331	
Developed straight length from the axis	l	0.2748893	m
Internal bend volume	V	0.001066985	m³
Internal bend volume	M	1.065071	kg
Relative roughness	$\bar{\Delta}$	0.0001422475	
Reynolds number	Re	90251	
Coefficient of local resistance	ζ_{loc}	0.1337544	
Roughness correction (Diagram 6-1)	k_{Δ}	1.010951	
Reynolds number correction factor (Diagram 6-1)	k_{Re}	1.340831	
Darcy Friction Factor	λ	0.01907611	
Coefficient of friction resistance	ζ_{fr}	0.07459201	
Pressure loss coefficient (based on the mean bend velocity)	ζ	0.2558981	
Hydraulic power loss	Wh	1.059659	W
Straight length of equivalent pressure loss	Leq	0.9430454	m

References:

[1] Handbook of Hydraulic Resistance, 3rd Edition, I.E. Idelchik

[2] Internal Flow System, Second Edition, D.S. Miller
